

# MINIMUM OVERHEAD BURST SYNCHRONIZATION FOR OFDM BASED BROADBAND TRANSMISSION

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## ABSTRACT

In this paper the subject of training data overhead in burst oriented transmission systems using OFDM is addressed. Previous work has shown that burst synchronization for such scenarios is possible. Alas the training data needed in some cases will significantly decrease the usable bandwidth. In order to minimize overhead we analyze the impact of synchronization errors on OFDM and derive a suitable optimization criterion. A novel training data format is introduced that allows to tailor the overhead according to the actual requirements. Using theoretical analysis and simulation the training data overhead can be decreased significantly without loss in performance.

## 1. INTRODUCTION

OFDM (*Orthogonal frequency division multiplexing*) is a well established technology for digital broadcasting applications [1]. Because of the numerous advantages of the OFDM principle there is a growing interest to use it for other applications [2, 3] as well. Some of the applications under consideration such as wireless ATM require a packet oriented transmission using short data bursts [4]. OFDM is well known to be vulnerable to synchronization errors. Therefore the requirements for the accuracy of synchronization units of the receiver are extremely high. They are further aggravated by the particularities of a burst oriented transmission:

1. The estimation of all relevant parameters must be established by the evaluation of a single preamble at a feasible complexity.
2. Whereas it is no big topic in a continuous transmission, in a scenario of very short bursts the training data needed for synchronization may reduce the usable bandwidth dramatically.

In the past several methods have been proposed that are capable to meet the requirements as far as accuracy and acquisition time are concerned [5, 6, 2, 7]. So far no

particular attention has been paid to the synchronization overhead which in some cases amounts to several OFDM symbols. It is obvious that for bursts consisting of few (possibly one) OFDM symbols this overhead cannot be accepted.

Therefore in this article we show how the synchronization overhead can be systematically reduced and adapted to the requirements of the application. In order to quantify the accuracy needed, we analyze the impact of synchronization errors on the OFDM signal. A novel frame format is presented which allows to tailor the overhead used for synchronization according to these requirements. Further analysis of suitable algorithms shows how much training data is in fact required.

## 2. SYSTEM MODEL

### 2.1. OFDM transmission over selective channels

Data symbols  $a_{l,k}$  are transmitted via an OFDM system using  $K_u$  subcarriers. Let  $l$  be the OFDM symbol number and  $k$  be the number of the subcarrier. The resulting signal is described by

$$s(t) = \sum_{l=-\infty}^{+\infty} \sum_{k=-K_u/2}^{K_u/2-1} a_{l,k} \Psi_{l,k}(t) \quad (1)$$

where  $\Psi_{l,k}(t)$  are the subcarrier pulses of length  $T_s = T_u + \Delta$ .

$$\begin{aligned} \Psi_{l,k}(t) &= e^{j2\pi \frac{k}{T_u}(t-\Delta-lT_s)} u(t-lT_s) \\ u(t) &= \begin{cases} 1 & 0 \leq t < T_s \\ 0 & \text{else} \end{cases} \end{aligned} \quad (2)$$

$T_u$  is the useful portion of the symbol which is periodically preceded by the guard interval of length  $\Delta$  to compensate for *inter symbol interference* (ISI). The transmitter is efficiently implemented using an IFFT of size  $N$ .

The signal is transmitted over a frequency selective channel

$$h(\tau, t) = \sum_i h_i(t) \cdot \delta(\tau - \tau_i) \quad (3)$$

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where  $h(\tau, t)$  comprises both the cir and the transmitter filter and has a maximum dispersion of  $\tau_{max} \leq \Delta$ . The received signal sampled at time instants  $T = T_u/N$  is given by

$$r(n) = \sum_i h_i(nT) \cdot s(nT - \tau_i) + n(nT) \quad (4)$$

where  $n(nT)$  denotes sampled white Gaussian noise of variance  $\sigma_N$ . We will further assume that  $E\{|a_{l,k}|^2\} = E\{|h_i|^2\} = 1$  and the signal to noise ratio averaged over all subcarriers is

$$E_s/N_0 = \frac{1}{\sigma_N} \quad (5)$$

Taking all this into account the received subcarrier symbols obtained via the FFT are described by

$$z_{l,k} = a_{l,k}H_{l,k} + n_{l,k} \quad (6)$$

where  $n_{l,k}$  can be shown to be white complex Gaussian noise and

$$H_{l,k} = \sum_i h_i(t) e^{2\pi\tau_i \frac{k}{T_u}} \quad (7)$$

## 2.2. Transmission Model and Receiver Tasks

Equation (6) applies only under assumption of ideal synchronization of transmitter and receiver. In general this will not be the case: The sampling time  $T'$  of the receiver will be different from  $T$ . The same holds for the carrier frequency leading to a frequency deviation of  $\Delta f$ . In addition a symbol timing offset of  $n_\epsilon \cdot T'$  has to be considered.

The resulting transmission model comprising all these effects is given by [8, 5]:

$$r(t_n) = e^{j2\pi\Delta f nT} \sum_i h_i(nT') \cdot s((n - n_\epsilon)T' - \tau_i) + n(nT') \quad (8)$$

In the receiver the deviations have to be estimated and compensated to an extent where they have no influence on the BER. As main synchronization tasks symbol timing synchronization, sampling clock synchronization and frequency synchronization can be identified.

In order to determine the importance of the single tasks, and to quantify the accuracy needed, the impact of synchronization errors on the OFDM signal must be analyzed.

## 3. IMPACT OF SYNCHRONIZATION ERRORS

### 3.1. Symbol Timing Errors

Identifying the 'beginning' of an OFDM symbol and correctly removing the guard interval is the first task to be performed. The remaining  $N$  samples are fed into the FFT for demodulation. Due to the guard interval some timing offset can be tolerated as long as the samples within the FFT window are influenced by only *one transmitted*

symbol. The subcarrier symbol disturbed by an offset of  $n_\epsilon$  samples can be described by [6]:

$$z_{l,k} = e^{j2\pi k \frac{n_\epsilon}{N}} \frac{N - n_\epsilon}{N} a_{l,k} H_{l,k} + n_{l,k} + n_{n_\epsilon}(l, k) \quad (9)$$

where  $n_\epsilon(l, k)$  can be assumed to be additional noise with power

$$P_{n_\epsilon} = \sum_i |h_i(t)|^2 \left( 2 \frac{\Delta \epsilon_i}{N} - \left( \frac{\Delta \epsilon_i}{N} \right)^2 \right) \quad (10)$$

and

$$\Delta \epsilon_i = \begin{cases} n_\epsilon - \frac{\tau_i}{T} & n_\epsilon T > \tau_i \\ \frac{\tau_i - \Delta}{T} - n_\epsilon & 0 < n_\epsilon T < -(\Delta - \tau_i) \\ 0 & \text{else} \end{cases}$$

This additional noise results from ISI and from *inter channel interference* (ICI) caused by the loss of orthogonality of the signal.

### 3.2. Frequency and Sampling Clock Errors

A static frequency offset  $\Delta f$  will lead to a phase rotation of the subcarrier symbols and to additional ICI. The received symbols are described by [9]

$$z_{l,k} = e^{j2\pi\Delta f l(T_s)} a_{l,k} si(\pi\Delta f T_u) H_{l,k} + n_{l,k} + n_{\Delta f}(l, k) \quad (11)$$

where  $n_{\Delta f}(l, k)$  can be assumed to be noise with power

$$P_{\Delta f} \approx \frac{\pi^2}{3} (\Delta f T_u)^2 \quad (12)$$

Taking into account the typical accuracy of oscillators frequency synchronization is mandatory for OFDM receivers..

Considering a relative sampling-clock offset of

$$\gamma = \frac{T' - T}{T} \quad (13)$$

the impact on the received subcarrier symbols is

$$z_{l,k} = e^{j2\pi k \gamma l \frac{T_k}{T_u}} a_{l,k} si(\pi k \gamma) H_{l,k} + n_{l,k} + n_\gamma(l, k) \quad (14)$$

where  $n_{\Delta f}(l, k)$  again is additional noise with power

$$P_\gamma \approx \frac{\pi^2}{3} (k\gamma)^2 \quad (15)$$

Comparing these results to equations (11) and (12) we see that the effects are comparable to those caused by a frequency offset  $\Delta f_k = k \cdot \gamma$  increasing with the subcarrier index  $k$ . In addition the symbol timing position will change. Under wireless transmission conditions  $P_\gamma$  can usually be neglected. No explicit sampling clock synchronization is needed.

#### 4. REQUIREMENTS OF A BURST ORIENTED TRANSMISSION

The ultimate requirement for the frame structure of a system (and the synchronization algorithms implied by this structure) is that the system performance should never be bounded by the achievable quality of the synchronization. In order to systematically minimize the required overhead we need to specify an adequate criterion to measure the impact of the synchronization on the system performance.

From the analysis in the previous section we see that all synchronization errors basically have two consequences:

1. A phase rotation of the received subchannel symbols.
2. Additional noise due to loss of signal power but more importantly due to loss of orthogonality.

The phase rotations cannot be distinguished from those caused by the channel transfer function. Hence they will be compensated in the same way by either a channel estimation or by the use of a differential modulation scheme. If we attribute the synchronization effects to a resulting transfer function  $H_{l,k}^R$  the following modified system model applies:

$$z_{l,k} = a_{l,k}H_{l,k}^R + n_{l,k} + n_s(l,k) \quad (16)$$

where  $n_s(l,k)$  is the additional noise caused by all synchronization errors with power

$$P_{sync}(m) = E\{|n_{\Delta f}|^2\} + E\{|n_{\gamma}|^2\} + E\{|n_{ne}|^2\} \quad (17)$$

and we assume that a synchronization parameter is held constant during a burst with burst number  $m$ . Thus the task of the receiver is to reduce  $P_{sync}(m)$  to an extent where its impact on decoding is negligible. In a transmission scenario with small bursts we can assume that the energy of the channel is constant for the duration of one burst. If the power of the signal at the receiver is held constant by a gain control the average power of the channel noise  $P_C(m)$  will change for every burst  $m$  and we define:

$$P_C(m) = E\{|n(l,k)|^2\} \quad (18)$$

The synchronization effects are negligible if the power of  $n_s(l,k)$  is significantly smaller than the power of the channel noise. This intuitive requirement can be expressed as follows:

$$P_{sync}(m) \leq \frac{1}{K}P_C(m) \quad (19)$$

The implicit bound for the accuracy of the receiver components given by (19) is more appropriate than the usual requirement that the synchronization error has to be within

certain limits. It implicitly takes into account, that if the SNR of a specific burst is too low for reliably decoding the data, additional noise caused by synchronization errors may be quite large too. Thus the absolute deviations may be quite large without degrading the overall system performance.

We define a synchronization failure as the event that the limit defined by (19) is exceeded:

$$P_f = P[P_{sync}(m) > 1/K \cdot P_C(m)] \quad (20)$$

The choice of  $K$  and  $P_f$  depends on the modulation and coding used and on the frame loss rate tolerable by the application. Having decided for specific values the synchronization overhead can now be systematically dimensioned.

#### 5. FRAME FORMAT

An efficient and reliable burst synchronization requires the use of training data dedicated to synchronization [8] (For methods relying solely on the evaluation of the guard interval as in [10, 11], correct synchronization cannot be guaranteed in the case of ISI). Generally this training data can be provided in the frequency domain, embedded into the subcarrier symbols, or in the time domain as explicit sequence. The sole use of frequency domain training data is problematic as far as the system acquisition is concerned: Due to the lack of orthogonality at the beginning of the synchronization procedure the subcarrier symbols are subject to sever ICI. In order to achieve the necessary accuracy the synchronization must be performed iteratively which contradicts the requirement of an instantaneous acquisition. Therefore this option will not be treated any further. Apart from explicit training data as

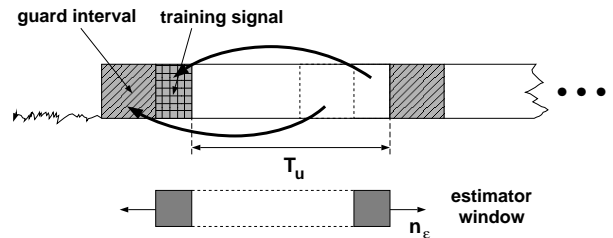


Figure 1: Frame format for burst synchronization

in [7, 12] periodic random data may be applied [6]. All these methods have in common that periodic signals are evaluated for synchronization. Most methods proposed use complete OFDM symbols as training data. This limits the choice of the frame format and one may be forced to use much more overhead than actually required. We therefore propose the frame format depicted in figure 1 which allows to freely choose the amount of symbol energy used for synchronization [13]. The first symbol of

every burst is not only preceded by the guard interval but also by an *additional* periodic part of  $L_S$  samples (The remaining symbols of the burst are still only preceded by the usual guard interval). Thus we exploit the periodicity inherent in every OFDM symbol and are able to efficiently synchronize even bursts consisting of *single* symbols.  $L_S$  can be chosen strictly according to the requirements of the system. For sampling-clock synchronization and the detection of large frequency offsets additional training data can be inserted at subcarrier level [7].

## 6. OPTIMIZATION OF THE FRAME STRUCTURE

We will further regard the worst case of a burst consisting of a single OFDM symbol. As example we will assume the transmission of a broad-band signal of 8 Mhz bandwidth over a GSM hilly-terrain channel as given in [6] using an OFDM system with  $N = 1024$ . For the assumed accuracy of the sampling clock oscillators no synchronization of this parameter is necessary even for high  $E_s/N_0$  of more than 20 dB.

### 6.1. Frequency Synchronization

A frequency offset can be divided into an offset of multiples of the subcarrier spacing  $1/T_u$  and a fractional offset of  $\varphi$ .

$$\Delta f = \frac{\varphi}{2\pi T_u} + \frac{k_\varepsilon}{T_u} \quad (21)$$

The orthogonality is only disturbed by the fractional offset  $\varphi$ . An integer portion will only lead to a shift of all subcarriers which must be detected. Since this can be done reliably with only few additional synchronization symbols [6, 7, 5] we will focus on the more critical task of estimating the fractional portion. An estimate for  $\varphi$  can be obtained by averaging over the phase differences of the periodic portions of the signal [14, 11]

$$\hat{\varphi} = \arg\left\{ \sum_{i=0}^{L_S-1} r(n+i+N) \cdot r^*(n+i) \right\} \quad (22)$$

Assuming that the estimate is Gaussian distributed its variance can easily be shown to be

$$\sigma_\varphi \approx \frac{1}{N_P \cdot E_s/N_0} \quad (23)$$

Though in the strict sense this result applies only for the AWGN channel it also proves to be accurate for frequency selective channels. Since the frequency offset can never be compensated completely, from equation (12) we determine an allowable maximum  $\varphi_{max}$  for the residing frequency offset  $\varphi_{res}$  after correction with the estimate obtained via equation (22). The probability that we fail to

meet this requirement can be derived via the tail probability of a Gaussian distribution with variance  $\sigma_\varphi$ :

$$P[\varphi_{res} > \varphi_{max}] \approx \frac{1}{\sqrt{2\pi}} \frac{\sigma_\varphi}{\varphi_{max}} e^{-\frac{\varphi_{max}^2}{2\sigma_\varphi^2}} \quad (24)$$

$$\approx \frac{1}{2} e^{-\frac{\varphi_{max}^2}{2\sigma_\varphi^2}} \quad (25)$$

Using (12) and (20) we can express the probability for a synchronization failure as a function of the synchronization overhead  $L_S$ . Using the approximation (25) we obtain:

$$P_f = e^{-\frac{6L_S}{K}} \quad (26)$$

Note that equation (26) is independent of both  $E_s/N_0$

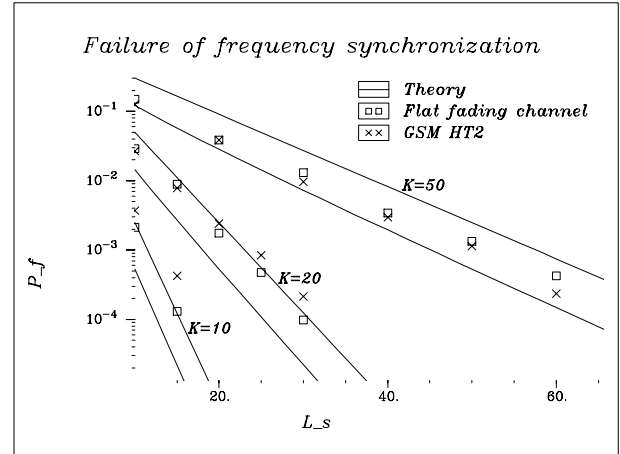


Figure 2: Probability of a failure of the frequency synchronization as function of the synchronization overhead.

and  $N$ . The accuracy of this result can be seen in figure 2 where we compare analysis with a stochastic simulation (The two theoretical curves are due to the two approximations for the tail probability). For flat fading channels the simulation results are well within the theoretical bounds. For selective channels the approximations necessary to obtain equation (23) are too optimistic. Nevertheless the synchronization error still has the property of being independent of the SNR. One can draw the following conclusions for the design of the frame structure:

1. By providing a certain  $L_S$  one can always guarantee a certain quality of the frequency estimate *independently* of the momentary SNR.
2. Since the choice of  $L_S$  is independent of  $N$  there are no restrictions as far as the number of subcarriers is concerned.

### 6.2. Timing-/Frame-Synchronization

Since at the beginning of the synchronization procedure we have to assume the frequency offset to be unknown,

timing synchronization via correlation is the most feasible alternative [14, 6, 7, 13]. The metric used in [14] is given by

$$\Lambda(n_t) = \sum_{i=0}^{L_S-1} |r(n_t+i+N)|^2 + |r(n_t+i)|^2 - 2 \left| \sum_{i=0}^{L_S-1} r(n_t+i+N) \cdot r^*(n_t+i) \right| \quad (27)$$

From this metric the correct starting point of the symbol can be estimated via

$$\min_{n_t} (\Lambda(n_t)) \rightarrow \hat{n}_e \quad (28)$$

The metric given in (27) has a fundamental advantage compared to the common correlation metric as given in [7]. The size of the correlation window  $L_S$  can be arbitrarily small whereas the correlation metric requires a large  $L_S$  to average out the effects of random data.

The error probability for the symbol estimation cannot be derived with passable effort. For evaluation we therefore rely on simulation: In analogy to [13] for random channels, distributed according to the channel profile under consideration, the symbol offset is estimated using the proposed algorithm. From the residing offset after correction with the estimated value the additional noise is calculated using (10) and the synchronization failure is determined according to (20).

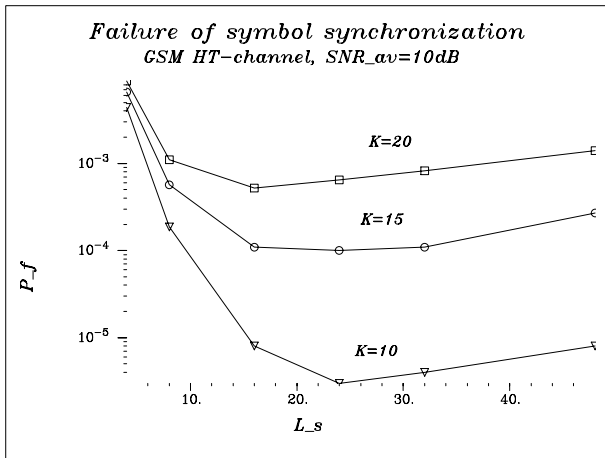


Figure 3: Probability of a failure of the symbol synchronization for an average SNR of 10dB.

The results obtained are quite different from those of the frequency estimation:

- The performance of the algorithm is no longer independent of the  $E_S/N_0$ . A comparison of figure 3 and 4 shows that though the absolute accuracy of the estimate will be better for a high SNR the impact on the received signal is less for the low SNR case.

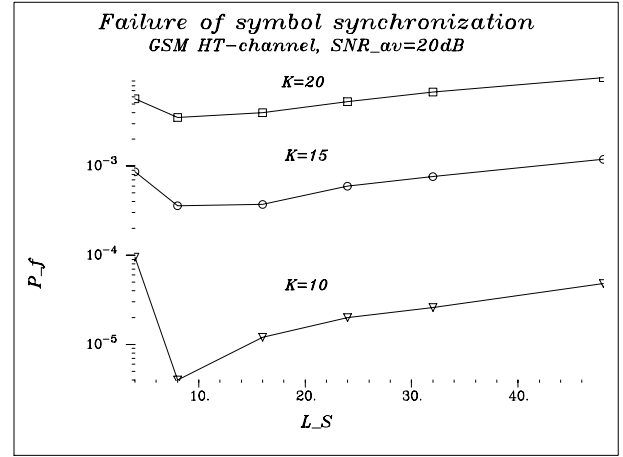


Figure 4: Probability of a failure of the symbol synchronization for an average SNR of 20dB.

- The accuracy of the synchronization cannot be increased beyond a certain limit even if  $L_S$  is further increased. For large values of  $L_S$  it even *decreases*. The minimum is reached for rather small values of  $L_S$ . Thus for high SNR cases there may be a trade-off between the quality of frequency and symbol synchronization.

The mechanism behind this behavior can be explained with a close analysis of the metric and will be subject of further publications. For many applications the achievable performance of the algorithm will suffice. Alas for high synchronization requirements combined with a low frame loss rate, symbol synchronization may turn out to be the system bottleneck. This may be resolved in several ways:

- Inserting multiple training sequences and averaging over several metrics. This of course will conflict with the concept of a single symbol burst
- The use of a priori information about the channel.
- Applying more sophisticated (though more complex) methods for symbol synchronization [13, 15].

## 7. RESULTS AND CONCLUSION

For the application example under consideration we can conclude the following: Since a burst oriented scenario will usually not be a high SNR scenario we see that with a choice of  $K = 10$ ,  $L_S \approx 25$  may already be sufficient. In this case  $P_f(m)$  will be in the same order of magnitude for both frequency and symbol synchronization at a low level of  $10^{-6}$  (For the channels under consideration one can expect the decoding of a burst to fail more frequently). Even if we consider the additional training data needed to

detect a shift of the subcarriers, we still achieve a considerable reduction of synchronization overhead compared to [2, 7].

As far as the performance of the synchronization algorithms is concerned the following was observed:

1. For frequency synchronization the probability of a synchronization failure is independent of the  $E_b/N_0$  and thus of fading effects. A certain quality can always be guaranteed, if sufficient training data is provided.
2. The common correlation based symbol synchronization scheme shows a bottoming of the performance if a too large number of samples are within the correlator window. For very large values of  $L_S$  the performance will even decrease. This implies that performance for some systems may be limited by the accuracy of symbol synchronization.

As long as the accuracy of the correlation based timing synchronization suffices the burst format presented here allows synchronization with minimum overhead. If higher accuracy is needed this can be achieved with the use of a single-carrier training sequence as proposed for frequency synchronization in [12]. Applying the results of [15] these sequences can also be used for symbol synchronization enabling powerful and implementationally simple algorithms. The use of these algorithms is at the cost of an increased synchronization overhead since the length of the synchronization sequence needed amounts to multiples of the guard interval.

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