The Safety Analysis: Disagreement of Wireless Communication-based Consensus

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***Abstract*—This paper considers the recent and practically relevant problem of safety in cyber-physical systems (CPS). Practical applications of CPS can be designed through the use of wireless multi-agent systems (MAS) since they provide flexibility and interoperability of the system functions. On the other hand, the performance of wireless MAS is prone to jamming attacks and the destructive effects of wireless channels. The aim of this paper is to determine safety bounds on wireless CPS under jamming attacks in presence of additive white Gaussian noise (AWGN) and wireless fading channels. The impact of both wireless channel impairments and jamming attacks on the performance of wireless MAS has been investigated for the first time. The disagreement vector, defined as the deviation from the desired consensus value of MAS based on the quantization error, is analyzed and investigated for AWGN and fading channels. Theoretical results are supported by matching simulation results.**

***Index Terms*—Cyber-physical system, consensus, jamming, multi-agent system, quantization error.**

I. Introduction

THE increasing demand for an integration of computation and networking has brought many new application fields for cyber-physical systems (CPS). By controlling physical entities through collaborating components, CPS bring together physical and virtual elements. In future CPS, control problems will be solved in a distributed manner over wireless communication channels to allow increased mobility, a wider coverage and reduced installation/operational costs when compared to the conventional wired solutions [1]–[3]. A typical application emerges in mobile scenarios where vehicles move autonomously. These vehicles will be controlled via wireless communication links. Another application area is industrial production. Machines or robots in shop-floors will be controlled via wireless links through control signals. The wireless transmission is an enabler for these future applications, however, due to the shared nature of the wireless communication channel, it also causes potential risks.

A multi-agent system (MAS) is an essential part of CPS which targets consensus as a global mission. Each agent has a limited knowledge and transmission capacity about the system's goal, can however interact with other agents to succeed the target mission like average consensus, maximum consensus, or minimum consensus [4]. Hence, a MAS has the potential to solve problems that are difficult for a single agent, while reducing the computational complexity and costs. An exemplary MAS model with four agents, $n = 4$, is given in Fig. 1, where n indicates the number of agents in the system. The four agents in the example aim at reaching average consensus in presence of a jammer node.

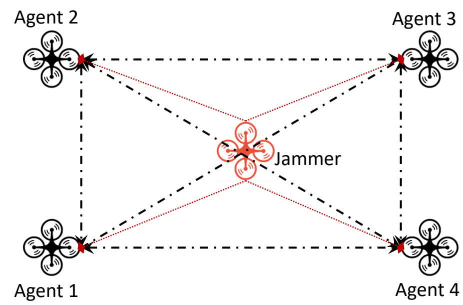

Fig. 1: The communication network of a wireless MAS with a jammer node.

The consensus problem of MAS for single-integrator agents is handled in various aspects as fixed/switching topologies and with/without delay by using algebraic graph theory, matrix theory, and control theory [4]. The MAS concept is first considered for wired scenarios, which assume error-free communication [5]. Although wireless MAS has the potential to be used in various application areas, the performance analysis of MAS in wireless networks is not a well-studied problem. The error-free transmission assumption, which is frequently resorted in the related literature of wired networks, is not valid for wireless channels. The case of Bernoulli distributed packet loss has been investigated in the average consensus problem in [6]. However, the Bernoulli distribution is not a realistic model to represent the wireless channel impairments.

Additionally, unlike wired counterparts, the transmission capability of wireless networks is restricted by the capacity of the channel due to the limited bandwidth of the channel. Agents have to quantize the continuous valued variables before sending the related variables to other agents. Hence, quantization error naturally arises due to the difference between the actual physical quantities and their quantized values [7], [8]. In [9], a capacity analysis of MAS is presented. An adaptive quantization protocol is also proposed in [9], based on the channel quality values. An event-based control strategy is proposed in [10], [11] for the average consensus problem in multi-agent systems in presence of limited capacity channels. These studies investigate noncontinuous transmission among agents. There

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are various triggering conditions, which ensure asymptotic convergence of the average consensus problem. The above-mentioned works do not investigate the transmission safety and the consensus of wireless MAS.

In addition to the wireless channel impairments and quantization errors, a wireless MAS is also vulnerable to jamming attacks due to the broadcast nature of the wireless channel. In this work, the destructive effect of jamming attacks on the average consensus problem is analyzed by using the disagreement vector as a performance monitoring tool. The main contributions of the paper are listed as:

- A methodology is introduced to combine various types of destructive effects which are system limitations (bandwidth and power), channel impairments (additive white Gaussian noise (AWGN) and fading components), and jamming effect into a single performance observation tool, referred to as the disagreement vector analysis.
- An upper bound on the disagreement vector of a wireless networked MAS is derived in presence of a jammer node.
- The derived theoretical expressions are verified by extensive simulation results.

II. Network Model and Consensus Analysis

In this section, a generalized MAS model with the consensus protocol used and algebraic graph theory definitions are given. The network model of any given MAS can be constructed according to the graph that indicates connections between agents. The graph, $\mathcal{G}$ consists of vertices (agents) and edges (the connections among agents), expressed as $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V}$ and $\mathcal{E}$ represent the set of agents and the set of all edges, respectively. $\mathcal{V}$ and $\mathcal{E}$ are defined as $\mathcal{V} = \{\mathrm{v}_1, \ldots, \mathrm{v}_n\}$ and $\mathcal{E} = \{\mathrm{e}_{ij} \mid i, j = 1, \ldots, n, i \neq j\}$. $\mathcal{A}$ defines the weighted adjacency matrix of the given network where $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n \times n}$. The cardinalities of the sets $\mathcal{V}$ and $\mathcal{E}$ are given as $|\mathcal{V}| = n$ and $|\mathcal{E}| = m$. An edge e_{ij} connects an agent represented by v_i to another agent v_j. In wireless networks, the gain of the channel between any two agents is modeled as a stochastic process due to the random nature of the transmission medium. The status of e_{ij} can be determined through the following formula

$$\mathrm{e}_{ij} = \begin{cases} 1, & \text{if } \gamma_{ij} > \gamma_{th} \\ 0, & \text{o.w.} \end{cases} \tag{1}$$

where γ_{ij} and γ_{th} represent the instantaneous signal to noise ratio (SNR) and the required threshold SNR, respectively. $\mathcal{D}$ is the diagonal degree matrix that contains the number of connected edges of each vertex in the system. The Laplacian matrix of $\mathcal{G}$ is calculated by

$$\mathbf{L}(\mathcal{G}) = \mathcal{D} - \mathcal{A}. \tag{2}$$

The second smallest eigenvalue of $\mathbf{L}(\mathcal{G})$, denoted by $\lambda_2(\mathcal{G})$ represents algebraic connectivity of $\mathcal{G}$, and determines the convergence speed of the consensus control [4].

In a MAS, each agent has distinct state variables, represented by $x_i(t) \in \mathbb{R}$. The state vector is represented by $\mathbf{x}(t)$ and is defined as $\mathbf{x}(t) = [x_1(t), \ldots, x_n(t)]$. The states of agents might represent physical quantities such as position, temperature, attitude, or voltage. The consensus of a MAS implies reaching an agreement on the states of the agents, which can be expressed as $x_i = x_j, \forall\, i, j,\ i \neq j$. The agents try to agree upon a specified system function such as average consensus, maximum consensus, or minimum consensus. In order to reach an agreement on the state values of all agents, in each time instant t, agent i sends its corresponding state value $(x_i(t))$ to other agents. Each transmission is realized in orthogonal time slots, hence, interference can be negligible in the system model used. In this work, the average consensus problem of MAS is investigated in wireless channels. The agents in the considered MAS have single integrator dynamics

$$\alpha = \sum_{i=1}^{n} x_i(0)/n, \tag{3}$$

where α represents the intended consensus value. To achieve average consensus, the network graph, $\mathcal{G}$, must be strongly connected and balanced for directed graphs [4]. In case of undirected topology, connectedness property is a sufficient condition to reach average consensus. The considered MAS consisting of single integrator with the dynamics,

$$\dot{\mathbf{x}}(t) = \mathbf{u}(t) \tag{4}$$

where $\mathbf{u}(t) = [u_1(t), \ldots, u_n(t)]^T$. The consensus protocol per agent is given as

$$u_i(t) = \sum_{j=1}^{n} a_{ij} \left(x_j(t) - x_i(t) \right), \tag{5}$$

and asymptotically solves the average consensus problem where $t \to \infty, x_i(t) \to \alpha = \sum_{i=1}^{n} x_i(0)/n$. Hence, the average of all states in time remains constant. The used consensus model proposed in [4] can be represented in compact notation as

$$\dot{\mathbf{x}}(t) = -\mathbf{L}(\mathcal{G}) \cdot \mathbf{x}(t). \tag{6}$$

A complete graph topology which guarantees to achieve the average consensus, is used to model the network. The states $\mathbf{x}(\mathrm{t})$ can be decomposed into two parts $\alpha = \frac{1}{\mathrm{n}} \mathbf{1}^{\mathrm{T}} \mathbf{x}(0)$, which is the constant mean value of $\mathbf{x}(t)$ over time, and a disagreement vector $\boldsymbol{\delta}(t)$ [4]. Thus, the state vector $\mathbf{x}(t)$ can be given as

$$\mathbf{x}(t) = \alpha \mathbf{1} + \boldsymbol{\delta}(t). \tag{7}$$

The disagreement vector, $\boldsymbol{\delta}(t)$ is a measure of group disagreement in the network and has zero average $(\mathbf{1}^T \boldsymbol{\delta}(t) \equiv 0)$ deduced from the definition [4].

Wireless channels restrict the transmission capability of the system depending on the channel quality. In order to analyze consensus problems in wireless media, we need to jointly consider control theory and information theory. Considering a digital wireless communication system, the agents can exchange their states only with a limited accuracy due to quantization of sensor readings. In this case, instead of using $x_i(t)$, each agent can capture the quantized value of $x_i(t)$ represented by $Q_i(t)$, $Q_i(t) = x_i(t) + e_i(t)$, where $e_i(t)$ is the quantization error. Hence, each agent exchanges the linear combination of quantized measurement values as

$$\begin{aligned} u_i(t) &= \sum_{j=1}^{n} a_{ij} \left(Q_j(t) - Q_i(t) \right) \\ &= \sum_{j=1}^{n} a_{ij} \left(x_j(t) + e_j(t) - x_i(t) - e_i(t) \right), \end{aligned} \tag{8}$$

where $Q_j(t)$ is the quantized value $x_j(t)$. Hence, we obtain the following expression

$$\mathbf{u}(t) = -\mathbf{L}(\mathcal{G}) \left(\mathbf{x}(t) + \mathbf{e}(t) \right), \tag{9}$$

where $e(t) = [e_1(t), \ldots, e_n(t)]^T$.

In order to assess quantization error levels, first the capacity analysis of wireless links is considered. The achievable rate of the channel is upper bounded by the channel capacity $R_{ij} \leq C_{ij}$, where $C_{ij} = \log(1+\gamma_{ij})$. Hence, R_{ij} determines the number of bits, denoted by N_{ij}, that can be exchanged over e_{ij}. Consequently, the maximal quantization error of a linear uniform quantizer for a given data rate R_{ij} is given by $\Delta_{ij} = q_i/2^{R_{ij}+1}$, where the quantization interval is defined as $q_i = \max_t(x_i(t)) - \min_t(x_i(t))$. We fix q_i for all agents, hence, we consider $q_i = q_m$ case. Accordingly, we can rewrite the quantization error in terms of the instantaneous SNR. With $\Delta_{ij}(\gamma_{ij}) = q_m/2^{N_{ij}+1}$, we obtain

$$\frac{q_m}{2\Delta_{ij}(\gamma_{ij})} = 2^{N_{ij}} \Leftrightarrow 1+\gamma_{ij} = \frac{q_m}{2\Delta_{ij}(\gamma_{ij})}. \tag{10}$$

Consequently, the instantaneous worst case quantization error is given by

$$\Delta_{ij}(\gamma_{ij}) = \frac{q_m}{2(1+\gamma_{ij})}. \tag{11}$$

In AWGN channel, the worst case quantization error expression is simplified to

$$\varepsilon(\bar{\gamma}_{ij}) = \frac{q_m}{2(1+\bar{\gamma}_{ij})}, \tag{12}$$

where $\bar{\gamma}_{ij}$ is the average SNR of the corresponding communication link. Considering fading channel, the worst case quantization error is calculated through the following formula

$$\varepsilon(\bar{\gamma}_{ij}) = \int \Delta_{ij}(\gamma_{ij}) f_\gamma(\gamma_{ij}) d\gamma_{ij} \tag{13}$$

where $f_\gamma(\gamma_{ij})$ indicates the probability density function (pdf) of γ_{ij}. Considering the frequently used Rayleigh fading channel model, the instantaneous SNR, γ_{ij}, is exponentially distributed. As a result of the integration in (13), we obtain the following expression for Rayleigh fading channels with unit power

$$\varepsilon(\bar{\gamma}_{ij}) = \frac{-q_m \exp(\bar{\gamma}_{ij}^{-1})}{2\bar{\gamma}_{ij}} \mathrm{Ei}\left(-\frac{1}{\bar{\gamma}_{ij}}\right), \tag{14}$$

where $\mathrm{Ei}(\cdot)$ denotes the exponential integral function. The performance of a wireless MAS system under jamming attacks will be presented in the next section.

III. Disagreement Analysis of Wireless Multi-Agent System under Jamming Attack

In this section, the disagreement vector analysis of MAS under jamming attacks are presented for both AWGN and Rayleigh fading channels. An example of MAS with four agents $n = 4$ with a jammer node in the communication network, is depicted in Fig. 1.

In the first analysis, AWGN channel is considered for both communication links among agents and jamming links between jammer node and agents of the MAS. The signal-to-interference-plus-noise ratio (SINR) of the link e_{ij} will be $\gamma'_{ij} = P_{ij}/(P_{J_i}+\sigma^2)$ where P_{ij} and P_{J_i} represents the received power of the agent i from the corresponding communication link and the jamming link, respectively. From (11), the instantaneous worst case quantization error of the communication link between agent i and j can be obtained as:

$$\Delta_{ij}(\gamma'_{ij}) = \frac{q_m}{2}\frac{1}{1+\dfrac{P_{ij}}{P_{J_i}+\sigma^2}} = \frac{q_m}{2}\frac{P_{J_i}+\sigma^2}{P_{J_i}+\sigma^2+P_{ij}}. \tag{15}$$

Let us define the SNR of the corresponding communication link and jamming link $\bar{\gamma}_{ij} = P_{ij}/\sigma^2$ and $\bar{\beta}_{J_i} = P_{J_i}/\sigma^2$,

$$\varepsilon(\bar{\gamma}_{ij}, \bar{\beta}_{J_i}) = \frac{q_m}{2}\frac{\bar{\beta}_{J_i}+1}{\bar{\beta}_{J_i}+\bar{\gamma}_{ij}+1}. \tag{16}$$

In the absence of a jammer node ($\bar{\beta}_{J_i} = 0$), (16) turns into (12). If both the communication channels and jamming channels are modeled as Rayleigh fading channel with unit power, the quantization error expression can be given as:

$$\begin{aligned}\varepsilon(\bar{\gamma}_{ij}, \bar{\beta}_{J_i}) =& \frac{q_m \exp(\bar{\gamma}_{ij}^{-1})(\bar{\beta}_{J_i} - \bar{\gamma}_{ij} - \bar{\gamma}_{ij}\bar{\beta}_{J_i})\mathrm{Ei}(-\bar{\gamma}_{ij}^{-1})}{2(\bar{\gamma}_{ij}-\bar{\beta}_{J_i})^2} \\ &+ \frac{q_m\bar{\beta}_{J_i}\left(\bar{\gamma}_{ij}\exp\left(\bar{\beta}_{J_i}^{-1}\right)\mathrm{Ei}\left(\bar{\beta}_{J_i}^{-1}\right)+\bar{\beta}_{J_i}-\bar{\gamma}_{ij}\right)}{2(\bar{\gamma}_{ij}-\bar{\beta}_{J_i})^2}\end{aligned} \tag{17}$$

by using the formula in (13). It is assumed that all communication and jamming links are uncorrelated.

The disagreement analysis of average consensus model is obtained by using algebraic graph theory and the quantization error expressions. Based on (7) and (9), the disagreement dynamics are defined by [10]

$$\dot{\boldsymbol{\delta}}(t) = -\mathbf{L}(\mathcal{G})(\boldsymbol{\delta}(t)+\mathbf{e}(t)). \tag{18}$$

Now, we will prove that $x(t)$ converges to $\alpha\mathbf{1}$ and $\boldsymbol{\delta}(t)$ is bounded for large t. The solution of the first order differential equation given in (7) satisfies [10]

$$\begin{aligned}\|\boldsymbol{\delta}(t)\| \leq& \exp(-\lambda_2 t)\|\boldsymbol{\delta}(0)\| \\ &+ \int_0^t \exp(-\lambda_2(t-\tau))\|\mathbf{L}(\mathcal{G})\cdot\boldsymbol{e}(\tau)\|d\tau,\end{aligned} \tag{19}$$

where $\lambda_2(\mathcal{G})$ denotes the second smallest eigenvalue of the Laplacian matrix of the graph where the minimum eigenvalue $\lambda_1(\mathcal{G}) = 0$. Also, $\|\boldsymbol{e}(\tau)\| \leq \sqrt{n}\varepsilon(\bar{\beta}_J^*, \bar{\gamma}_{ij})$, and $\|\mathbf{L}(\mathcal{G})\boldsymbol{e}(\tau)\| \leq \|\mathbf{L}(\mathcal{G})\|\cdot\|\boldsymbol{e}(\tau)\|$ where $\bar{\beta}_J^* = \max_i \bar{\beta}_{J_i}$. Then,

$$\begin{aligned}\|\boldsymbol{\delta}(t)\| \leq& \exp(-\lambda_2 t)\|\boldsymbol{\delta}(0)\| \\ &+ \int_0^t \exp(-\lambda_2(t-\tau))\|\mathbf{L}(\mathcal{G})\|\sqrt{n}\varepsilon(\bar{\beta}_J^*, \bar{\gamma}_{ij})d\tau.\end{aligned} \tag{20}$$

Hence,

$$\begin{aligned}\|\boldsymbol{\delta}(t)\| \leq& \exp(-\lambda_2 t)\|\boldsymbol{\delta}(0)\| \\ &+ \frac{\sqrt{n}\cdot\varepsilon(\bar{\gamma}_{ij}, \bar{\beta}_J^*)\|\mathbf{L}(\mathcal{G})\|(1-\exp(-\lambda_2 t))}{\lambda_2}.\end{aligned} \tag{21}$$

For $t \to \infty$, $\|\boldsymbol{\delta}(t)\|$ is upper bounded as

$$\|\boldsymbol{\delta}(\infty)\| \leq \|\boldsymbol{\delta}_{\boldsymbol{u}}\| = \frac{\sqrt{n}\cdot\varepsilon(\bar{\gamma}_{ij}, \bar{\beta}_J^*)\|\mathbf{L}(\mathcal{G})\|}{\lambda_2}. \tag{22}$$

To obtain the bound disagreement expression, $\|\boldsymbol{\delta}_{\boldsymbol{u}}\|$, (16) or (17) are substituted in (21) under AWGN and Rayleigh fading channels, respectively. The topology factor $\phi(\mathcal{G})$ represents the limit value of the worst case result of the disagreement while $\bar{\beta}_J^* \to \infty$. Hence, the topology factor can be defined as

$$\phi(\mathcal{G}) = \frac{\sqrt{n}\|\mathbf{L}(\mathcal{G})\|q_m}{2\lambda_2}. \tag{23}$$

In a complete graph $\|\mathbf{L}(\mathcal{G})\|$ is equal to n. $\mathbf{L}(\mathcal{G})$ always has a zero eigenvalue where the corresponding eigenvector is a vector of all ones, $[1,\ldots,1]^T$. The second smallest eigenvalue is given by $\lambda_2 = n$ for complete graphs. Consequently, for the fully connected networks the limit value of the topology factor can be simplified as

$$\phi(\mathcal{G}) = \phi(n) = \frac{q_m\sqrt{n}}{2}. \tag{24}$$

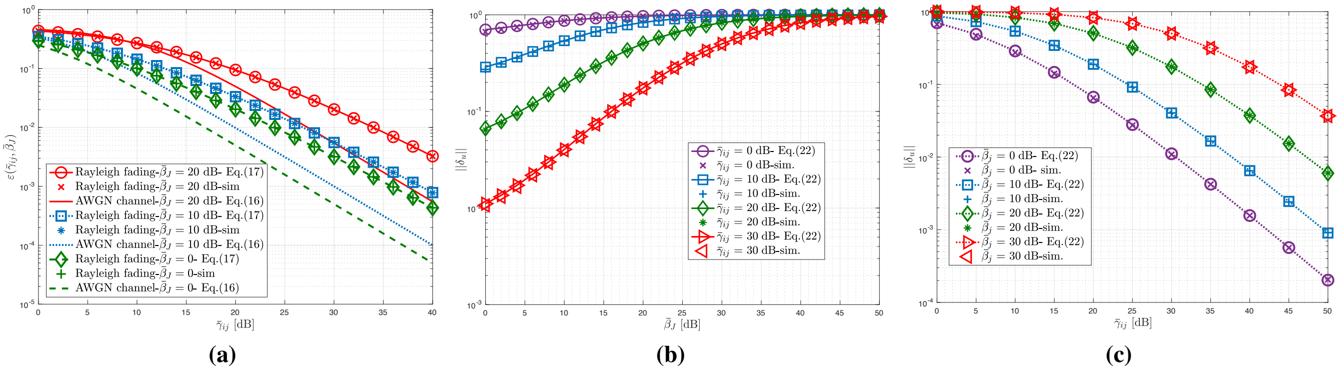

Fig. 2: (a) The quantization error results of the given network versus varying $\bar{\gamma}_{ij}$ with and without a jammer node are presented in both AWGN and Rayleigh fading channels. (b)With respect to $\bar{\gamma}_{ij}$ and $\bar{\beta}_J$, the simulation and bounded disagreement results of the given network with a jammer node are presented in Rayleigh fading channels. (c) The simulation and bounded disagreement results of the given network with a jammer node are given versus various $\bar{\gamma}_{ij}$ values in Rayleigh fading channels.

IV. Numerical Results

In this section, the calculations of theoretical results supported with the corresponding simulation results are presented. The Monte Carlo simulations are performed for the generation of channel coefficients. Extensive numerical results are provided to detail the theoretical derivations. To simplify the perception of the figures, it is assumed that the average SNR values of the communication channels among agents are the same. The average SNR values of the communication channels between agents and the jammer node are also kept symmetric $\bar{\beta}_{J_i} = \bar{\beta}_J$. The quantization interval is equal to $q_m = 1$ in all scenarios.

The effect of SNR of agents and jammer node on the quantization error, $\varepsilon\left(\bar{\gamma}_{ij}, \bar{\beta}_J\right)$ is investigated in different scenarios as Rayleigh fading channel assumption, AWGN channel assumption and no jammer case. The numerical results are shown in Fig. 2(a). The first observation is that the best result is obtained in the case of no jammer node in AWGN channel, as expected. The increase in the SNR of jammer node, $\bar{\beta}_J$ results in higher quantization error levels for all channel cases. The fading channel also deteriorates the quantization error performance. There is approximately 7 dB difference between the error performances of AWGN and Rayleigh fading at the error level of 10^{-2}.

Fig. 2(b) depicts the upper bound on disagreement vector ($\|\boldsymbol{\delta_u}\|$, in (22)) as a function of the $\bar{\beta}_J$ for different $\bar{\gamma}_{ij}$ values in Rayleigh fading channels. The increase in average SNR of agents improves the error performance for all cases. When $\bar{\beta}_J \rightarrow \infty$, $\|\boldsymbol{\delta_u}\|$ converges to $q_m\sqrt{n}/2$, as easily deduced from (22). For this setup, $q_m\sqrt{n}/2$ is equal to 1, which is also shown in the figure.

The last figure, Fig. 2(c) demonstrates how the average SNR of the jammer node affects the upper bound on the disagreement in Rayleigh fading channels. For all $\bar{\beta}_J$ values, $\|\delta_u\|$ decreases while $\bar{\gamma}_{ij}$ increases. For high $\bar{\gamma}_{ij}$ value, $\bar{\beta}_J$ also has significant influence on $\|\delta_u\|$. For example, when $\bar{\gamma}_{ij} = 40$ dB, $\|\delta_u\|$ is equal to 0.5×10^{-3} and 2×10^{-1} in cases of $\bar{\beta}_J = 0$ dB $\bar{\beta}_J = 30$ dB, respectively. Hence, the SNR of the jammer node needs to be considered in the MAS design to ensure the reliability of the system.

V. Conclusion

In this paper, we have obtained an upper bound expression on the disagreement vector of MAS for both AWGN and Rayleigh fading channels in case of jamming attack. We have validated our results with extensive simulation results. We have demonstrated that the jamming attack causes a bottleneck on the disagreement performance of wireless MAS. We have determined the performance limits of the reliable working range of MAS, which gives the institution about the implementation requirements of MAS. For a future work, a task is to design robust control systems along with considering efficient multiple access techniques such as non-orthogonal transmission schemes against jamming attacks.

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