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Application of Graph Theory to the Multicell Beam Scheduling Problem

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***Abstract*—This paper applies combinatorial optimizations techniques to improve the downlink transmission to multiple users in a network with N cells where intercell interference is a performance limiting factor. A fair distribution of the signal-to-interference-plus-noise ratio (SINR) is desirable. A well known technique to get a fair (balanced) SINR is max–min unicast beamforming (MBF). However, in a multicell network there are conditions where MBF can result in a low balanced SINR. This happens if, e.g., two users are geographically close together and served by two base stations from, e.g, two different interfering sectors. Then, the mutual interference among the two links will be large and the balanced SINR among all jointly optimized links decreases. Therefore, the users must be scheduled such that these situations are avoided.**

The smart selection of active beams to avoid intercell interference is called beam scheduling in this paper and leads to a combinatorial optimization problem. This paper proposes a graph theory based problem that is closely related to the beam scheduling problem. The proposed algorithms maximize the sum rate or the minimum SINR among all users and slots. For the two-cell case an optimal algorithm exists. In the $N > 2$-cell case, the beam scheduling problem is proved to be $\mathcal{NP}$-hard. Based on the close relation between the beam scheduling problem and the multidimensional assignment problem, this paper presents suboptimal algorithms for $N > 2$ to maximize either the sum rate or the minimum SINR among all users and slots. The performance gain in terms of the mean sum rate or the minimum SINR is significant compared to random scheduling.

***Index Terms*—Beamforming, beam scheduling, assignment problem, bottleneck assignment problem**

I. Introduction

IN multicell networks, intercell interference limits performance if the frequencies are reused in adjacent cells. An unfair distribution of the signal-to-interference-plus-noise ratio (SINR) or a low overall sum rate may result. On the other hand, frequency reuse factors larger than one reduce the spectral efficiency. Upcoming standards as Long-Term Evolution (LTE)-advanced desire a high spectral efficiency. Therefore, fully coordinated networks with a single frequency band are promising to increase the overall system throughput. New standards as LTE-advanced allow the use of multiple antennas techniques to mitigate intercell interference. Using multiple smart antennas at the base stations (BSs), a famous technique to improve the fairness among users is max–min beamforming (MBF).

However, MBF has the drawback of a low overall sum rate in some cases. An unfavorable scheduling decision can decrease the sum rate performance. Such a situation is caused by a beam-collision. If two users are closely located and served by different BS arrays and their antenna array (beamforming) vectors are optimized with MBF, a low balanced SINR among the users can be the result. Consequently, different and jointly served links should be spatially separated, otherwise mutual interference can decrease the overall sum rate. The avoidance of these unfavorable scheduling decisions is called beam-scheduling problem. The combination of MBF and beam scheduling may result in a fair distribution of the SINR and jointly in a sufficiently large sum rate of the system.

A. Scenario

This paper considers a fully coordinated multicell network as depicted in Fig. 1. Each of the N base station (BS) arrays uses N_A correlated antenna elements to form beam lobes in the direction of the scheduled users. The transmit power of each antenna element is limited to a power constraint of P_C. Each user has a single antenna element. Coordination among a large number of BSs will be difficult, if instantaneous channel state information (CSI) [1], [2] is used, due to the increased backhaul effort. However, using long-term CSI this effort is reduced [1]. For transmit beamforming, this statistical CSI is a practically relevant information to form beam lobes in the direction of users, while considering the interference of adjacent cells [3]. Based on long-term CSI, this paper proposes a technique for the joint optimization of the beamforming vectors, the power control, and the beam scheduling along with multiuser scheduling.

B. Related work

Intercell interference mitigation based on multicell transmit beamforming with joint power control has been investigated intensively during the last ten years, [1], [4]–[6]. Coordinated max–min beamforming achieves a fair balanced SINR among all jointly scheduled users in a network while the transmit power per antenna element or per antenna array is limited to a power constraint [7], [8]. Iterative low complexity algorithms for the MBF with per-antenna power constraints are proposed in [7]–[9]. Hence, the joint optimization of transmit power and beamforming vectors enables a fair distribution of the SINR

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among a set of scheduled users. A drawback of MBF is the low sum rate that results in case of unfavorable scheduling decisions. Consequently, a smart scheduling of the users is required.

The work [10] discusses the information theoretical aspects of multiuser scheduling and beamforming. The authors show that in environments with slow fading, the diversity gain can be improved with multiuser scheduling along with opportunistic beamforming.

A problem related to beam scheduling is called channel assignment problem (CAP). In addition to the exploitation of multiuser diversity, interference mitigation is another important issue, especially in multicell networks. The assignment of channels to users or cells also influences the interference in a network. In [11], the authors propose the CAP for cellular networks. The aim was to allocate a number of channels to cells such that certain constraints are satisfied.

Another possibility to improve the SINR is an optimized beam scheduling, since beam lobes cause interference in adjacent sectors [12]. A first simple round-robin beam switching approach to avoid these beam collisions is proposed in [13]. The articles [12], [14] propose optimized approaches which take into account the channel quality information of the users or the geographical data. However, these approaches consider only small, e.g, 2-cell scenarios and do not jointly optimize the beamforming vectors. The work [15] considers a spatial scheduling with the objective to cancel the interference by, e.g. zero-forcing. The previous work [16] uses max–min fairness among the users as objective. In contrast to [12], [14], [15], the work [16] uses long-term CSI in the form of spatial correlation matrices to optimize the beam scheduling, beamforming, and user scheduling jointly. The MBF optimization results in a balanced SINR and these SINR values are used to compute a cost function for the beam scheduling problem.

In contrast to [16], where the balanced mean SINR is used for the objective function, the authors in [17] use so-called interference constraints to define groups of beams which are mutually interfered with each other. Hence, instead of the SINR optimization, this scheme can be seen as an orthogonal approach of finding non-interfering beams such that the overall performance is maximized.

Instead of an interference avoidance, the authors of [18] propose a max–min fair antenna assignment scheme for a system with geographically dispersed antenna ports. The selection of antenna ports is optimized such that the SINR of the weakest user is maximized. Instead of a simple optimization of the assignment of stations (ports) to users [18], this paper proposes optimization techniques to optimize the beamforming vectors along with the temporal scheduling of users such that the sum rate, or the minimum SINR is maximized. This paper shows the close relationship between the multicell beam scheduling problem and the $\mathcal{NP}$-hard multidimensional assignment problem (MAP) [19], [20]. A straightforward approach for solving the MAP is simulated annealing (SA) [21]. In addition to SA, this paper proposes further algorithms with better performance.

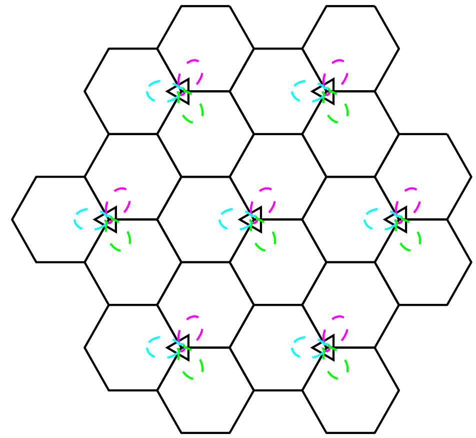


Fig. 1: Scenario: The lobes show the orientation of the antenna patterns. The triangles denote BSs. Each BS has three sectors. All BSs are coordinated to enable a joint beamforming and beam scheduling.

C. Contributions

This paper is based on the previous work [16] and presents the following contributions:

- Our previous work [16] uses the sum rate as an objective function to get a higher system throughput. This approach is reasonable for applications as, e.g., internet downloads. However, in several applications, e.g, video conferences, also fairness is desired, then the sum rate approach could be not the best choice. Therefore, this paper presents a novel problem formulation based on a graph theoretical max–min problem which achieves a more fair distribution of the mean SINR over the time. Previous works as [12], [14], [15], [17] use predefined beamformers or zero-forcing to avoid interference by a beam selection. This paper uses the balanced SINR of a max–min beamformer as cost function for a temporal user scheduling. The result is an avoidance of a low balanced SINR which is a known drawback of MBF.
- The schemes in [12], [14], [15], [17], [18] use instantaneous channel quality information (CQI). Furthermore, the optimization is performed only for one time instant. This paper generalizes this approach and optimizes the beam scheduling over the stationary interval of the long-term statistics of the channel [3]. Therefore, this paper considers in addition to SINR fairness also temporal fairness.
- This paper presents a graph theoretical framework for the N-cell beam scheduling problem and proves the $\mathcal{NP}$-hardness of the problem. The earlier work [16] presents a suboptimal approach for the N-cell scenario. This paper additionally shows that the general beam scheduling problem in the 2-cell scenario is equivalent to a linear assignment problem which can be optimally solved by a well known polynomial time algorithm.
- The N-cell beam scheduling problem has a close relation with the multi-dimensional assignment problem (MAP). Several heuristics can be used to find good

suboptimal solutions. One commonly used approach for $\mathcal{NP}$-hard problems is SA. This paper presents a simple SA based algorithm which finds good solutions for the beam scheduling problem. SA is a randomized heuristic. Additionally, this paper proposes alternative deterministic dimension-wise optimization methods based on an extension of the optimal approaches for the 2-cell scenario. The different methods are compared in terms of complexity, sum rate performance, and max–min SINR fairness.
- In extension to the sum rate maximization approaches, this paper shows a low complexity greedy algorithm with less than half of the complexity compared to the other algorithms. Additionally, the new approach can guarantee a higher temporal fairness. Instead of a so-called quasi-fair scheduling (QFS), where each user is scheduled equally often, the greedy approach can guarantee so-called opportunistic Round-Robin scheduling (ORRS) fairness which is a temporal fair scheduling scheme (further details are introduced in Section III-B1). Table I depicts an overview of all proposed algorithms.
- Finally, a detailed analysis of all proposed algorithms (depicted in Table I) concerning complexity, temporal fairness, SINR fairness, and sum rate is presented.

D. Outline

Section II presents the system setup and the data model of the investigated system depicted in Fig. 1. Section II-A defines the MBF problem and the beam scheduling problem and proves the $\mathcal{NP}$-hardness of the general $N > 2$-cell beam scheduling problem. Section III presents optimization techniques to solve the beam scheduling problem. For $N = 2$ an optimal solution can be computed in polynomial time. Two approaches, a fair and a sum rate maximizing approach, are presented in Section III-A. Section III-B presents four heuristics for the general N-cell case. Each algorithm has different advantages, e.g., low complexity, temporal fairness, SINR fairness, or high sum rate. Section IV gives a comparison of the proposed algorithms and highlights the advantages of each algorithm. This paper concludes with a summary and a short discussion in Section V.

E. Notation

Lower case and upper case boldface symbols denote vectors and matrices respectively. The nth element of a vector is denoted with $[\mathbf{a}]_n$. The element with indices n, m of a matrix $\mathbf{A}$ is denoted with $[\mathbf{A}]_{n,m}$. The mth column vector of a matrix $\mathbf{A}$ is denoted with $[\mathbf{A}]_{:,m}$. Respectively, $[\mathbf{A}]_{n,:}$ denotes the nth row vector. The conjugate transpose of a matrix $\mathbf{A}$ is denoted with $\mathbf{A}^H$. In $\mathbf{a}_k^t$ t and k denote indexes. Finally, $\mathrm{LCM}\{a, b\}$ denotes the lowest common multiple of a and b.

II. System setup and data model

Regard a network with N cooperative BS arrays as depicted in Fig. 1. In this network, M users are equally distributed. A BS schedules one user per cell, hence, in a scheduling slot N users are active.

The matrix $\mathbf{S} \in \mathbb{N}^{N\times K}$ defines the assignments of these users to BSs and scheduling slots with index k. Each element in $\mathbf{S}$ with index c, k is given by

$$[\mathbf{S}]_{c,k} = i;\;\; \text{user } i \text{ is scheduled by BS } c \text{ in slot } k \qquad (1)$$

The integer K denotes the total number of orthogonal slots. Here, orthogonal means orthogonal in the temporal domain or orthogonal scheduling slots. However, an application to channels orthogonal in the frequency domain is also feasible. An extension to a multi-carrier system is straightforward. Let $\mathbf{C}_c \in \mathbb{N}^{K_T\times K_F}$ denote the matrix of all K_T orthogonal time and K_F frequency slots of a BS c, then $\mathbf{s}_c = \mathrm{vec}(\mathbf{C}_c) \in \mathbb{N}^{1\times[K_T\cdot K_F]}$ denotes the vectorized version of this matrix and corresponds to one row of the matrix $\mathbf{S}$.

A user $i = [\mathbf{S}]_{c,k}$, equipped with one antenna element, in cell c receives from its BS array of the cell in slot k the signal

$$r_{i,k} = \mathbf{h}_{i,i}^H \boldsymbol{\omega}_{i,k} s_i + \sum_{l\in[\mathbf{S}]_{:,k},\, l\neq i} \mathbf{h}_{l,i}^H \boldsymbol{\omega}_{l,k} s_l + n_i. \qquad (2)$$

The vector $\mathbf{h}_{l,i} \in \mathbb{C}^{N_A\times 1}$ is the multiple input single output (MISO) channel between the BS antenna array serving user l and user i. Each BS antenna array serving a user i uses beamforming matrices $\boldsymbol{\omega}_{i,k} \in \mathbb{C}^{N_A\times 1}$ to form a beam lobe to user i in slot k. The scalar n_i denotes the interference plus noise of adjacent networks with the assumption $\mathbb{E}\{|n_i|^2\} = \sigma_i^2$ and $\mathbb{E}\{n_i\} = 0$. The desired signal transmitted to user i is denoted with s_i with $\mathbb{E}\{|s_i|^2\} = 1$ and $\mathbb{E}\{s_l s_i^*\} = 0$ if $l \neq i$. With the instantaneous downlink SINR

$$\hat{\gamma}_{i,k} = \frac{|\mathbf{h}_{i,i}^H \boldsymbol{\omega}_{i,k}|^2}{\sum_{\substack{l\in[\mathbf{S}]_{:,k}\\ l\neq i}} |\mathbf{h}_{l,i}^H \boldsymbol{\omega}_{l,k}|^2 + \sigma_i^2}, \qquad (3)$$

the achievable rate a user i achieves is given by

$$\hat{R}_{i,k} = \mathbb{E}\{\log(1 + \hat{\gamma}_{i,k})\}. \qquad (4)$$

A global optimization based on instantaneous CSI over a large number of slots in a large network with multiple users and BSs is very difficult. An optimization based on the long-term CSI is practically more relevant in a multicell scenario. Therefore, as in [22], the ergodic capacity is approximated by

$$\hat{R}_{i,k} = \mathbb{E}\{\log(1 + \hat{\gamma}_{i,k})\} \approx \log(1 + \gamma_{i,k}) = R_{i,k}. \qquad (5)$$

The variable $\gamma_{i,k}$ denotes the mean SINR

$$\gamma_{i,k} = \frac{\boldsymbol{\omega}_{i,k}^H \mathbf{R}_{i,i} \boldsymbol{\omega}_{i,k}}{\sum_{\substack{l\in[\mathbf{S}]_{:,k}\\ l\neq i}} \boldsymbol{\omega}_{l,k}^H \mathbf{R}_{l,i} \boldsymbol{\omega}_{l,k} + \sigma_i^2} \qquad (6)$$

with the definition of the spatial correlation matrices

$$\mathbf{R}_{l,i} = \mathbb{E}\{\mathbf{h}_{l,i}\mathbf{h}_{l,i}^H\}. \qquad (7)$$

The rate $R_{i,k}$ is an approximation of the achievable rate. This paper uses the rate $R_{i,k}$ as a performance measure. All optimizations of the beamforming vectors $\boldsymbol{\omega}_{l,k}$ and the scheduling decisions are made based on the long term CSI

TABLE I: Overview of the different algorithms

Algorithm	Temporal fairness	Objective	Approach	Complexity
A1: random RRS	RRS	-	random solution	low
A2: dim.-wise sum rate maximization	QFS	sum rate	dimension-wise optimization	high
A3: dim.-wise max–min optimization	QFS	max–min SINR	dimension-wise optimization	high
A4: greedy sum rate maximization	ORRS	sum rate	greedy algorithm	low
A5: sum rate based SA	QFS	sum rate	randomized local search	high

given in (7). For a fixed beamforming strategy (in this paper MBF), the total sum rate over all scheduling slots K

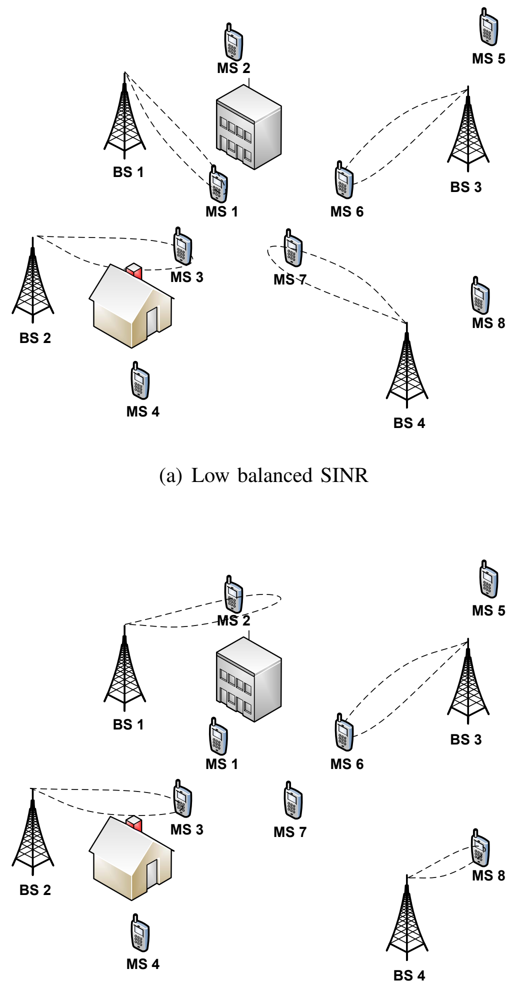
$$R(\mathbf{S}) = \sum_{k=1}^{K} \sum_{i \in [\mathbf{S}]_{:,k}} \log \left(1 + \frac{\boldsymbol{\omega}_{i,k}^H \mathbf{R}_{i,i} \boldsymbol{\omega}_{i,k}}{\sum_{\substack{l \in [\mathbf{S}]_{:,k} \\ l \neq i}} \boldsymbol{\omega}_{l,k}^H \mathbf{R}_{l,i} \boldsymbol{\omega}_{l,k} + \sigma_i^2} \right) \tag{8}$$

is used as a cost function.

A. Problem Formulation

A promising method to mitigate interference among a set of scheduled users and to achieve fairness is MBF. The result is a balanced SINR among all jointly active users using the same channel resource. However, an unfavorable scheduling decision can result in a low balanced SINR among all jointly scheduled users. In what follows, a network with global knowledge of the long-term CSI is considered. The long-term CSI in the form of spatial correlation matrices (7) can be either transmitted to a central unit or by a distributed optimization at each BS. In the centralized approach, a central unit will then optimize the scheduling decisions and beamforming weights. In the distributed approach, the computation of the scheduling decisions and beamforming weights is performed locally at each BS. In this case, the long-term CSI must be forwarded to each cooperative BS. For both approaches, the scheduling decisions and the beamforming weights can be reused as long as the channel is stationary. Therefore, a low backhaul overhead is the result. Fig. 2(a) depicts an example with a low balanced SINR among the scheduled users. Users 1, 3, 6 and, 7 are jointly scheduled and located in the same geographical region. Therefore, all beams (denoted with dashed lobes) are directed in the same region. A consequence could be a low balanced SINR among the users. Fig 2(b) shows a better scheduling decision. In this figure, users 2, 3, 6, and 8 are jointly scheduled by their BSs. These users are located more distributed. Less mutual interference and, therefore, a higher SINR after the beamforming optimization can be the consequence. The optimization presented in this paper can be categorized in two parts. The first part is MBF (introduced in Section II-A1) which is used for the computation of a cost matrix for the beam scheduling problem. This section is followed by Section II-A2 which introduces the beam scheduling problem.

1) Beamforming Problem: It is desired to maximize the lowest SINR $\gamma_{i,k}$ of all jointly scheduled users $i \in [\mathbf{S}]_{:,k}$ in slot k, where the power of each antenna element is subject to a power constraint P_C. This problem can be formally expressed as a MBF problem with per-antenna array element power



(a) Low balanced SINR

(b) Higher balanced SINR

Fig. 2: Example: Beamforming based on scheduling decisions. A column (slot) in the scheduling matrix (1) in case (a) is $[\mathbf{S}]_{:,1} = [1, 3, 6, 7]^T$. In case (b) the scheduling decision is different and may result in less mutual interference. The corresponding column (slot) in the scheduling matrix is $[\mathbf{S}]_{:,1} = [2, 3, 6, 8]^T$.

constraints:

$$\gamma_k = \max_{\boldsymbol{\Omega}_k} \min_{i \in [\mathbf{S}]_{:,k}} \gamma_{i,k} \tag{9}$$
$$\text{s.t. } |[\boldsymbol{\omega}_l]_a|^2 \leq P_C \;\; \forall a \in \mathcal{A}_l, \; \forall l \in [\mathbf{S}]_{:,k}.$$

The index set of antenna array elements of the BS assigned to user i is denoted with $\mathcal{A}_i$. For simplification each BS antenna array uses the same number N_A of array elements. The matrix concatenated by the total set of beamforming vectors in slot k is denoted with $\boldsymbol{\Omega}_k = [\boldsymbol{\omega}_{1,k}, \ldots, \boldsymbol{\omega}_{N,k}]$. The problem (9) is non-convex in general. However, for special instances,

$$R_\Sigma = \max_{\boldsymbol{\pi}_2,\ldots,\boldsymbol{\pi}_N} \sum_{k=1}^{K} \sum_{i\in[[\boldsymbol{\pi}_1^T,\ldots,\boldsymbol{\pi}_N^T]^T]_{:,k}} \log\left(1+\frac{\boldsymbol{\omega}_{i,k}^H \mathbf{R}_{i,i}\boldsymbol{\omega}_{i,k}}{\sum_{\substack{l\in[[\boldsymbol{\pi}_1^T,\ldots,\boldsymbol{\pi}_N^T]^T]_{:,k}\\ l\neq i}} \boldsymbol{\omega}_{l,k}^H \mathbf{R}_{l,i}\boldsymbol{\omega}_{l,k}+\sigma_i^2}\right) \tag{10}$$

e.g., rank-1 spatial correlation matrices ($\mathrm{rank}(\mathbf{R}_{l,i}) = 1$) the problem (9) is quasi-convex [4], [5]. The authors of [5] present a globally optimal solution based on a second order cone problem [23]. Later, [9] introduces a direct and a low complexity solution.

2) Beam Scheduling Problem: A smart assignment of jointly scheduled users increases the balanced SINR which is the solution of the MBF problem (9). The main idea for the optimization presented in this paper is now the avoidance of unfavorable scheduling decisions such that the beamforming vectors can achieve a higher balanced SINR which results in a higher sum rate. The scheduling assignment is given by the matrix (1). Hence, matrix $\mathbf{S}$ is beside the beamforming vectors $\boldsymbol{\Omega}_k$ $k = 1,\ldots,K$ another optimization variable. To simplify the following notations, each cell contains exactly K active users. A straightforward optimization goal is the sum rate maximization of the approximated rate defined in (5). To define the sum rate maximization the following Definition is helpful:

Definition 1: Let $\boldsymbol{\pi}_1 = [i_1, i_2, \ldots i_K] \in \mathbb{N}^K$ be an index vector and let $\mathbf{P}_c \in \{0,1\}^{K\times K}$ be a permutation matrix, with $\sum_{l=1}^{K}[\mathbf{P}_c]_{n,l} = 1$ and $\sum_{l=1}^{K}[\mathbf{P}_c]_{l,n} = 1$, $\forall n = 1,\ldots,K$. A permutation $\boldsymbol{\pi}_c = \boldsymbol{\kappa}(\boldsymbol{\pi}_1)$ of a index-vector $\boldsymbol{\pi}_1 = [i_1, i_2, \ldots, i_K]$ is given by a permutation of the elements of the vector: $\boldsymbol{\pi}_c = \boldsymbol{\kappa}(\boldsymbol{\pi}_1) = \boldsymbol{\pi}_1\mathbf{P}_c$.

With the assumption of an equal number of users per cell and BS antenna array and with the set $\mathcal{W} = \{\boldsymbol{\Omega}_1,\ldots,\boldsymbol{\Omega}_K\}$ of all feasible beamforming matrices, the optimization problem can be stated as: Find the optimal permutations $\boldsymbol{\pi}_c$ of row vectors of the scheduling matrix $\mathbf{S}$ such that the sum rate is maximized. The permutation form of the scheduling matrix is $\mathbf{S} = [\boldsymbol{\pi}_1^T, \boldsymbol{\pi}_2^T, \ldots, \boldsymbol{\pi}_N^T]^T$. With the assumption of a fixed first permutation $\boldsymbol{\pi}_1$, the optimization of the scheduling matrix $\mathbf{S}$ is defined by finding optimal permutations of $\boldsymbol{\pi}_2,\ldots,\boldsymbol{\pi}_N$ of the row vectors $[\mathbf{S}]_{2,:},\ldots,[\mathbf{S}]_{N,:}$ such that (10) is maximized. Using long-term CSI in the form of spatial correlation matrices (7), the solution of problem (10) gives the matrix $\mathbf{S}$ for an optimized the beam scheduling. The beamforming vectors stored in matrix $\boldsymbol{\Omega}_k$ are optimized based on a MBF problem given by Eq. (9). The problem (10) matches to a problem of the graph theory:

Definition 2: Axial multidimensional assignment problem (MAP) [24]: Having an N-partite graph G with parts $X_1 = X_2 = \ldots = X_N = \{1,2,\ldots,K\}$, find a set of K disjoint cliques in G of the maximal total weight if every clique e_k in G is assigned a weight $w(e_k)$.

The axial MAP is $\mathcal{NP}$-hard. In [25], the author proves that the 3-dimensional ($N \geq 3$) axial assignment problem is $\mathcal{NP}$-hard. This is a special case of the MAP, hence the MAP is $\mathcal{NP}$-hard as well.

Proposition 1: Finding the optimal scheduling matrix maximizing the sum rate in problem (10) is $\mathcal{NP}$-hard.

Proof: The proof of the $\mathcal{NP}$-hardness is straightforward and is proven by the relation of the beam scheduling problem to the $\mathcal{NP}$-hard axial MAP. This problem given in Definition 2 directly maps to a special case of the beam switching problem (10). In the case of exactly K users per cell, the number of scheduling slots assigned to the users is K. Then, each cell c (row index of $\mathbf{S}$) corresponds to a dimension of a N-dimensional axial MAP with K elements per dimension. Note, each user is assigned exactly once to a scheduling slot. The goal is finding the optimal permutations of row vectors of the matrix $\mathbf{S}$, such that the costs (10) are maximized. The costs of scheduling decisions $[\mathbf{S}]_{:,k}$ correspond to the costs $w(e_k)$ of cliques e_k given by

$$w(e_k) := \sum_{i\in[[\boldsymbol{\pi}_1^T,\ldots,\boldsymbol{\pi}_N^T]^T]_{:,k}} R_{i,k}(\boldsymbol{\pi}_1,\ldots,\boldsymbol{\pi}_N)$$

with

$$R_{i,k}(\boldsymbol{\pi}_1,\ldots,\boldsymbol{\pi}_N) = \log\left(1+\frac{\boldsymbol{\omega}_{i,k}^H \mathbf{R}_{i,i}\boldsymbol{\omega}_{i,k}}{\sum\limits_{\substack{l\in[[\boldsymbol{\pi}_1^T,\ldots,\boldsymbol{\pi}_N^T]^T]_{:,k}\\ l\neq i}} \boldsymbol{\omega}_{l,k}^H \mathbf{R}_{l,i}\boldsymbol{\omega}_{l,k}+1}\right). \tag{11}$$

Thus, maximizing the sum rate of all slots k also solves the axial N-dimensional MAP with K elements per dimension. ■

Example 1: Regard the example in Fig. 2b). Assume the scheduling decision of a joint scheduling of users 2, 3, 6, and 8 in the first slot and the scheduling of users 1, 4, 5, and 7 in the second results in a maximized overall sum rate. The optimal scheduling matrix is then given by:

$$\mathbf{S}_{example} = \begin{bmatrix} 2 & 3 & 6 & 8 \\ 1 & 4 & 5 & 7 \end{bmatrix}^T. \tag{12}$$

Hence, there are $K = 2$ scheduling slots and $N = 4$ cells. This corresponds to a 4-partite graph with K elements. The problem of finding the maximum sum rate is equivalent to a 4-dimensional MAP of finding 2 disjoint cliques with 4 elements. Fig. 3 depicts the equivalent graph representation of the example.

III. Beam Scheduling Optimization

A. Optimal 2-Cell Scenario

To simplify the understanding of the investigations of the general N-cell scenario of Section III-B, this section investigates the simple 2-cell scenario at first. The 2-cell scenario is part of the genereal N-cell scenario depicted in Fig. 1 where only two adjacent cells are cooperative. The goal is to find optimal scheduling decisions in the adjacent cells such that

$$[\mathbf{W}]_{k_1(i_1),k_2(i_2)} = \log\left(1+\frac{\boldsymbol{\omega}_{i_1}^H\mathbf{R}_{i_1,i_1}\boldsymbol{\omega}_{i_1}}{\boldsymbol{\omega}_{i_2}^H\mathbf{R}_{i_2,i_1}\boldsymbol{\omega}_{i_2}+\sigma_{k_1}^2}\right)+\log\left(1+\frac{\boldsymbol{\omega}_{i_2}^H\mathbf{R}_{i_2,i_2}\boldsymbol{\omega}_{i_2}}{\boldsymbol{\omega}_{i_1}^H\mathbf{R}_{i_1,i_2}\boldsymbol{\omega}_{i_2}+\sigma_{i_2}^2}\right) \tag{13}$$

$$\begin{aligned}[\mathbf{W}]_{k_1(i_1),k_2(i_2)} = \max_{\boldsymbol{\Omega}} \quad & \min\{\frac{\boldsymbol{\omega}_{i_1}^H\mathbf{R}_{i_1,i_1}\boldsymbol{\omega}_{i_1}}{\boldsymbol{\omega}_{i_2}^H\mathbf{R}_{i_2,i_1}\boldsymbol{\omega}_{i_2}+\sigma_{i_1}^2},\frac{\boldsymbol{\omega}_{i_2}^H\mathbf{R}_{i_2,i_2}\boldsymbol{\omega}_{i_2}}{\boldsymbol{\omega}_{i_1}^H\mathbf{R}_{i_1,i_2}\boldsymbol{\omega}_{i_2}+\sigma_{i_2}^2}\} \\ \text{s.t.} \quad & |[\boldsymbol{\omega}_{i_1}]_a|^2 \le P_C \;\; \forall a \in \mathcal{A}_{i_1}, \\ & |[\boldsymbol{\omega}_{i_2}]_a|^2 \le P_C \;\; \forall a \in \mathcal{A}_{i_2}.\end{aligned} \tag{14}$$

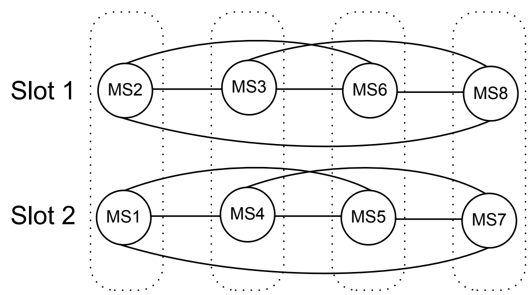


Fig. 3: Example of a scheduling graph with four cells and two slots. The users (mobile stations (MS)) are denoted by nodes (circles). The selected disjoint cliques are connected with edges. A part corresponds to a cell. Each cell contains two users.
Costs of clique 1:
$w(e_1) = \log(1+\gamma_{2,1})+\log(1+\gamma_{3,1})+\log(1+\gamma_{6,1})+\log(1+\gamma_{8,1})$
Costs of clique 2:
$w(e_2) = \log(1+\gamma_{1,2})+\log(1+\gamma_{4,2})+\log(1+\gamma_{5,2})+\log(1+\gamma_{7,2})$

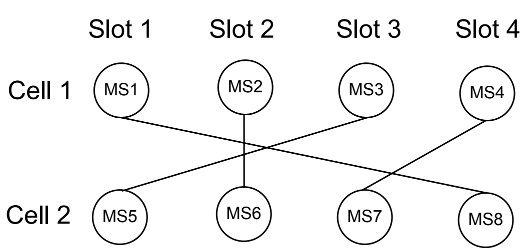


Fig. 4: Example of a 2-cell scheduling graph. The users are denoted by nodes (circles). The selected disjoint cliques are connected with edges. Each cell contains four users. Each clique corresponds to a scheduling slot. In this example $K = 4$ disjoint cliques are selected.

the balanced SINR is improved. This section discusses two approaches:

1) The first approach desires a maximized sum rate over all scheduling slots (see Section III-A1). This approach is useful for applications where a quality-of-service rate is not desired, e.g., internet downloads.
2) The second approach improves the weakest SINR over all scheduling slots. Consequently, the max–min fairness is further improved (see Section III-A2). The outcome of this approach is an increased worst SINR. Hence, this approach can be applied in applications where each user requires constantly the same rate, e.g, in video conferences.

These two approaches for the 2-cell beam scheduling problem are formulated based on a simple bipartite graph model. Assuming each cell contains K users. The users of each cell correspond to a part of the bipartite graph. The problem of beam scheduling is to find K pairs of users, where one user is selected from both cells, so that the objective (sum rate 1) or minimum SINR 2)) is maximized. Hence, K disjoint cliques in the bipartite graph must be selected. Fig. 4 presents an example for the 2-cell scenario. The following sections present two methods to optimize the beam scheduling problem according to the two presented objectives.

1) Linear Sum Assignment Problem: The first problem is to find the optimal scheduling matrix $\mathbf{S}$ such that the max–min fair beamforming problem (9) results in a higher sum rate. One advantage of the 2-cell scenario is the efficient algorithm which exists in this case. As already mentioned in Section II-A, the N-cell beam scheduling problem maps perfectly to the N dimensional assignment problem. In the 2-cell scenario, this problem corresponds to a linear sum assignment problem.

Definition 3: Linear sum assignment problem (LSAP): Having a bipartite graph G with parts $X_1 = \{1, 2, \ldots, K\}$ and $X_2 = \{1, 2, \ldots, K\}$, find a set of K disjoint cliques in G of the maximal total weight if every clique e_k in G is assigned a weight $w(e_k)$.

In Fig. 4, a bipartite graph depicts a possible assignment of a 2-cell example with $K = 4$ users per cell. Let $\mathbf{W}$ be a cost matrix with $[\mathbf{W}]_{k_1,k_2} = w_{k_1,k_2} \in \mathbb{R}_+$, and $\mathbf{X}$ be a matrix of assignments with $[\mathbf{X}]_{k_1,k_2} = x_{k_1,k_2} \in \{0, 1\}$ and let $x_{k_1,k_2} \in \{0, 1\}$ $k_1, k_2 = 1, \ldots, K$, the LSAP can be also modeled as

$$\begin{aligned}\max \quad & \overbrace{\sum_{k_1=1}^{K}\sum_{k_2=1}^{K} w_{k_1,k_2}x_{k_1,k_2}}^{=f_0(\mathbf{W},\mathbf{X})} \\ \text{s.t.} \quad & \sum_{k_2=1}^{K} x_{k_1,k_2} = 1 \;\; k_1 = 1, \ldots, K, \\ & \sum_{k_1=1}^{K} x_{k_1,k_2} = 1 \;\; k_2 = 1, \ldots, K.\end{aligned} \tag{15}$$

Let $k(i)$ denote the slot in which user i has been scheduled. The mapping $k(i)$ is bijective: there is exactly one user mapped to one slot. Regarding the notation of the LSAP presented in (15), the LSAP corresponds to the bipartite matching problem where the weights $w_{k_1(i_1),k_2(i_2)}$ of the edges among all disjoint node pairs of the two parts X_1 and X_2 are maximized. The variable $x_{k_1(i_1),k_2(i_2)}$ is equal to one if a user i_1 in slot $k_1(i_1)$ of cell 1 is jointly scheduled with a user i_2 in slot $k_2(i_2)$ of cell 2. The objective function $f_0(\mathbf{W},\mathbf{X})$ is then the sum rate of all user pairs i_1, i_2 of the slots $k_1(i_1)$, $k_2(i_2)$. Using MBF, the cost matrix for a maximized sum rate can be defined as in (13). To simplify the notation the index k is removed at the beamforming matrix and vectors. In this paper, the SINR fairness is desired and the solution of a MBF problem is used for the beamforming matrices $\mathbf{\Omega}$. The LSAP can be solved optimally in polynomial time. In [26], [27] the authors present the first polynomial time algorithm that computes the optimal solution of the LSAP based on a cost matrix $\mathbf{W}$. The so called Hungarian method [26] solves the LSAP in $O(K^4)$. Later, the work [28] presents an $O(K^3)$ implementation of the Hungarian method.

2) Linear Bottleneck Assignment Problem: Another objective function for the optimization of the scheduling matrix $\mathbf{S}$ is a further improvement of the fairness. The idea is to find scheduling decisions in the two cells such that the SINR of the weakest slot is maximized. This corresponds to an additional SINR balancing over all slots by an optimal assignment of jointly scheduled users. The linear bottleneck assignment problem (LBAP) of the graph theory matches perfectly to this approach. It has a similar linear programming formulation as the LSAP. The only difference is the objective function

$$f_0(\mathbf{W},\mathbf{X}) = \min_{1\leq k_1,k_2\leq K} w_{k_1,k_2} x_{k_1,k_2}. \tag{16}$$

The optimal value of the objective function is one of the coefficients $w_{k_1(i_1),k_2(i_2)}$ of the cost matrix. The result is an assignment such that the lowest costs are maximized. The MBF (9) directly delivers the coefficients of the cost matrix (14).

The LBAP can be solved with less complexity compared to the LSAP. In [20, page 174], the authors propose an algorithm that solves the LBAP with an $K \times K$ cost matrix $\mathbf{W}$ in $O(K^{2.5}\sqrt{\log(K)})$. To find the optimal solution of the LBAP, this paper uses the threshold algorithm presented in [20].

B. $N > 2$-Cell Scenario

The optimization of the beam scheduling problem in a $N > 2$-cell scenario is $\mathcal{NP}$-hard. No optimal polynomial time solution exists, otherwise $\mathcal{P} = \mathcal{NP}$. The beam scheduling problem is a graph theoretical problem. As explained in Section II-A2, the optimization of the scheduling matrix $\mathbf{S}$ corresponds to a MAP. An $N = 4$-cell scenario is presented in Example 1. Each cell containts $K = 2$ users. The optimization of the beam scheduling is to find $K = 2$ disjoint groups of users where exactly one user out of each cell is selected, such that a performance metric (e.g., sum rate) is maximized. This group of users is served together in a slot. The graph theoretic interpretation as an MAP is as follows: a slot corresponds to a clique in the N-partite graph. Each user i is a node in this graph. The set of users in a cell c corresponds to a part of nodes X_c. Hence, the problem is the search for K disjoint cliques in a N-partite graph, such that the costs of the each clique are maximized. The costs are given by the total sum rate (8). Using permutations $\boldsymbol{\pi}_1,\ldots,\boldsymbol{\pi}_N$ of the parts $X_1,\ldots,X_N$ of an N-partite graph, the permutation form of the MAP is given by [24]:

$$\max_{\boldsymbol{\pi}_2,\ldots,\boldsymbol{\pi}_N} \sum_{i=1}^{K} w(i,\boldsymbol{\pi}_2(i),\ldots,\boldsymbol{\pi}_N(i)). \tag{17}$$

Comparing (17) with (10) shows the perfect matching of the beam scheduling problem with the MAP. The MAP or the beam scheduling problem can be also seen as a generalized (multidimensional) LSAP

$$\begin{aligned}
\max \quad & \sum_{\substack{k_j=1\\ j=1,\ldots,N}}^{K} w_{k_1,\ldots,k_N} x_{k_1,\ldots,k_N} && (18)\\
\text{s.t.} \quad & \sum_{\substack{k_j=1\\ j=1,\ldots,N\ j\neq 1}}^{K} x_{k_1,\ldots,k_N} = 1, \quad k_1 = 1,\ldots,K, \\
& \cdots \\
& \sum_{\substack{k_j=1\\ j=1,\ldots,N\ j\neq N}}^{K} x_{k_1,\ldots,k_N} = 1, \quad k_N = 1,\ldots,K, \\
& x_{k_1,\ldots,k_N} \in \{0,1\} \quad k_1,\ldots,k_N = 1,\ldots,K
\end{aligned}$$

with a multidimensional cost matrix

$$\mathbf{W} \in \mathbb{R}_+^{\overbrace{K\times K\times\cdots\times K}^{N \text{ times}}}. \tag{19}$$

The axial MAP is $\mathcal{NP}$-hard, therefore, only suboptimal solutions are feasible. In the next sections four different heuristics are investigated.

In what follows, this paper proposes four algorithms, each having different advantages. Each algorithm outperforms the random Round-Robin scheduling (RRS). The comparison of the heuristics must be fair, therefore, the following section illustrates the investigated fairness constraints (QFS and ORRS) assumed in this paper. The algorithms are based on different search strategies. Therefore, Section III-B2 introduces the so-called dimensionwise permutation and the so-called p-exchange neighborhood. The section is followed by the description of each algorithm. Table I depicts an overview of all algorithms.

1) Scheduling Fairness: The previous sections always assume an equal number of active users per cell. This is an ideal scenario because usually the number of users is different among adjacent cells. In addition to a fair distribution of the mean SINR per user and slot, a fair allocation of the scheduling slots to users is desired. Let $\mathcal{U}_{c,0}$ be the set of $n_c = |\mathcal{U}_c|$ active users in cell c. To achieve an equal number of users per cell, in cells with less than K users, a user index can be inserted several times. However, all users must be scheduled equally often. To guarantee a fair allocation of the scheduling slots to users in each cell it is obvious that at least

$$K = \mathrm{LCM}\{n_1, n_2, \ldots, n_N\} \tag{20}$$

scheduling slots are needed. After K slots the scheduling matrix $\mathbf{S}$ and the set of beamforming vectors $\mathcal{W}$ can be reused as long as the long-term CSI is stationary.

Example 2: Consider a small network with a cell c_1 with users $\mathcal{S}_{c_1} = \{1,2,3\}$ and a cell c_2 with users $\mathcal{S}_{c_2} = \{4,5\}$. The number of scheduling slots to achieve a fair allocation of scheduling slots per user in a cell is $K = \text{LCM}\{2,3\} = 6$. Hence, a valid scheduling matrix could be

$$\mathbf{S}_1 = \begin{bmatrix} 3 & 3 & 2 & 2 & 1 & 1 \\ 4 & 4 & 5 & 4 & 5 & 5 \end{bmatrix}. \tag{21}$$

According to the scheduling matrix $\mathbf{S}_1$ in cell c_1, each user is scheduled twice and in cell c_2 each user is scheduled three times.

In some applications, besides the overall fair allocation of all scheduling slots to users, temporal fairness is also important. The delay between two consecutive transmissions should not be too large. This paper uses two definitions of temporal fairness:

Definition 4: In opportunistic Round Robin scheduling (ORRS), the transmission time of each BS is divided to $r_c = K/n_c$ rounds, whereby each user in the cell must be served once in every round without a fixed order. With this scheme, the maximum time interval between two consecutive slots allocated to a user equals to $2(n_c - 1)$ scheduling slots.

Definition 5: In quasi fair scheduling (QFS), the users can be arbitrarily assigned to the K time slots. With this scheme, the maximum delay between two consecutive transmissions to a user equals to $(n_c - 1)K/n_c$ time slots. This constraint is temporally unfair.

Example 3: The scheduling matrix (21) of example 2 satisfies the QFS criterion but violates the ORRS criterion, because, e.g, in the first round of cell c_1 the user 1 is not scheduled. The following permutation of row vectors of matrix $\mathbf{S}$ given by

$$\mathbf{S}_2 = \begin{bmatrix} 2 & 1 & 3 & 2 & 3 & 1 \\ 4 & 5 & 5 & 4 & 5 & 4 \end{bmatrix} \tag{22}$$

satisfies the ORRS fairness constraint.

2) Local Neighborhood and Solution Space: Many heuristics for the axial MAP rely on the so-called local neighborhood search. The heuristics in this paper use two different types of local neighborhoods:

1) Local neighborhood with an permutation in s dimensions
2) local p-exchange neighborhood

In the literature $p = 2$-exchange local neighborhoods are often used [29]. However, the size of the dimension-wise permutation neighborhood is larger. Therefore, a dimension-wise algorithm can possibly find better solutions than an algorithm based on a 2-exchange neighborhood. In what follows, different heuristic are proposed. At first, Section III-B3 introduces the first heuristic based on SA with a 2-exchange neighborhood. This section is followed by dimension-wise approaches (Section III-B4 and III-B5) outperforming the SA technique. The SA and the dimension-wise algorithms satisfy the QFS constraint. Therefore, Section III-B6 additionally presents a low complexity greedy approach that satisfies the ORRS temporal fairness constraint.

Algorithm 1 Sum rate maximization based on SA

```
Initialize: T_0, T := T_0, Create random solution S_0, S_best := S_0 , S_a := S_0
while T > 0 do
  Take a randomly a cell c
  Compute random neighboring solution : S_b := p̂(S_a, 2, c)
  Solve the MBF problem (9) with S = S_b → W
  With W compute (8) → R(S_b)
  if R(S_b) ≥ R(S_best) then
    S_best := S_b
  end if
  if R(S_b) ≥ R(S_a) then
    S_a := S_b
  else
    Generate random number r uniformly in the range (0, 1)
    if r ≤ Prob(T, R(S_a), R(S_b)) then
      S_a := S_b
    end if
  end if
  T := T − 1
end while
return S_best, W
```

3) Simulated Annealing based sum rate maximization: In [21], the authors propose a 2-exchange local search to solve a MAP. A similar approach is applied in this paper for the N-cell beam scheduling problem. The number of beamforming problems increases with the number of scheduling matrices. SA is a local search algorithm and has less computational complexity compared to, e.g., genetic algorithms where multiple solutions (scheduling matrices) are required. The algorithm presented in [21] is based on four simple steps and can be directly applied to the scheduling matrix $\mathbf{S}$:

1) Random selection of a cell c (dimension) of the initial matrix $\mathbf{S}_a$.
2) Random selection of two user indexes i and j of this cell c.
3) Compute $\mathbf{S}_b$ by exchanging the two indexes i and j in c.
4) If the new solution $\mathbf{S}_b$ has a higher sum rate (8) than $\mathbf{S}_\alpha$, then accept the new solution. The search is continued until some maximum number of iterations is reached.

A detailed outline of the final algorithm is presented in Alg. 1. The algorithm starts with an initial solution $\mathbf{S}_0$ and returns the best solution $\mathbf{S}_{best}$ that is computed during the whole search. In each iteration, two users are exchanged in two columns (slots). Consequently, two beamforming problems of a network with N users are solved. Local search algorithms can stay in local optima. To escape from these suboptimal solutions, an extension of the local search to a randomized algorithm like SA is useful. At the first iterations, the SA algorithm takes with some probability $Prob(T, R(\mathbf{S}_a), R(\mathbf{S}_b))$ a weaker solution $\mathbf{S}_b$ with $R(\mathbf{S}_b) \leq R(\mathbf{S}_a)$. With an increased number of iterations this probability decreases, such that at the end only the strong solutions are taken. To avoid the multiple computation of one

$$[\mathbf{W}]_{k,k_c(i_c)} = \sum_{i\in\{[\mathbf{S}]_{:,k}\cup i_c\}} \log(1+\frac{\boldsymbol{\omega}_i^H \mathbf{R}_{i,i}\boldsymbol{\omega}_i}{\sum_{\substack{l\in\{[\mathbf{S}]_{:,k}\cup i_c\}\\ l\neq i}} \boldsymbol{\omega}_l^H \mathbf{R}_{l,i}\boldsymbol{\omega}_l + \sigma_i^2}) \tag{23}$$

$$[\mathbf{W}]_{k,k_c(i_c)} = \max_{\boldsymbol{\Omega}} \min_{i\in\{[\mathbf{S}]_{:,k}\cup i_c\}} \frac{\boldsymbol{\omega}_i^H \mathbf{R}_{i,i}\boldsymbol{\omega}_i}{\sum_{\substack{l\in\{[\mathbf{S}]_{:,k}\cup i_c\}\\ l\neq i}} \boldsymbol{\omega}_l^H \mathbf{R}_{l,i}\boldsymbol{\omega}_l + \sigma_i^2} \tag{24}$$
$$\text{s.t. } |[\boldsymbol{\omega}_i]_a|^2 \leq P_C \;\; \forall a\in\mathcal{A}_i, \;\; \forall i\in\{[\mathbf{S}]_{:,k}\cup i_c\}$$

Algorithm 2 Dimension-wise sum rate maximization

Initialize: $\mathbf{S} = [\boldsymbol{\pi}_1^T]^T$
for $c = 2$ to N **do**
  Select an adjacent cell c with the users with index i_c stored in $\mathcal{U}_c$.
  for all slots $k_c(i_c) = 1, \ldots K$ of users with index i_c in cell c **do**
    for all slots $k = 1$ to K **do**
      Compute the beamforming matrices $\boldsymbol{\Omega}_k$ with (25)
      With $\boldsymbol{\Omega}_k$ determine the entry of the cost matrix (23)
    end for
  end for
  Compute the optimal assignment for $\mathbf{W}$ with a LSAP
  $\boldsymbol{\pi}_c \leftarrow$ LSAP.
  $\mathbf{S} = [\mathbf{S}^T, \boldsymbol{\pi}_c^T]^T$
end for
return $\mathbf{S}$,$\mathcal{W}$

Algorithm 3 Dimension-wise max-min optimization

Initialize: $\mathbf{S} = [\boldsymbol{\pi}_1^T]^T$
for $c = 2$ to N **do**
  Select an adjacent cell c with the users with index i_c stored in $\mathcal{U}_c$.
  for all slots $k_c(i_c) = 1, \ldots K$ of users with index i_c in cell c **do**
    for all slots $k = 1$ to K **do**
      Determine the entry of the cost matrix (24)
    end for
  end for
  Compute the optimal assignment for $\mathbf{W}$ with a LBAP
  $\boldsymbol{\pi}_c \leftarrow$ LBAP.
  $\mathbf{S} = [\mathbf{S}^T, \boldsymbol{\pi}_c^T]^T$
end for
return $\mathbf{S}$,$\mathcal{W}$

solution, the investigated SA heuristic never computes the same 2-exchange solution again. Algorithm 1 depicts the used SA heuristic.

4) Dimension-Wise Sum Rate Maximization: Compared to a p-exchange neighborhood, a dimension-wise permutation neighborhood is much larger and a heuristic using such a large neighborhood can achieve better solutions at the expense of an increased number of solutions which must be investigated. This section presents a heuristic based on a dimension-wise permutation heuristic for the N-cell beam scheduling problem based on the following observations:

- A user i in a cell c mainly receives interference from its two adjacent cells if a sector pattern as in Fig. 1 is used.
- In the case of $N = 2$, the MAP is a LSAP. In this case the MAP can be solved optimally in $O(K^3)$ as presented in Section III-A1.

Due to the pattern and the property that most of the interference is received from the two adjacent cells, a LSAP finds near optimal solutions. Hence, the algorithm must investigate a lower number of solutions. The dimension-wise heuristic combines the two observations to reduce the number of beamforming optimizations. The idea of the algorithm can be summarized in the following steps:

- The algorithm starts in cell $c = 1$ and selects an adjacent cell $c = 2$. For all users with index k_1 of cell $c = 1$ and all users with index k_2 of cell $c = 2$ a cost matrix $\mathbf{W}$ of a LSAP according to (14) is computed by solving the beamforming problem (9) for all user combinations. The first row of the final scheduling matrix is $\mathbf{S} = [\boldsymbol{\pi}_1^T]^T$. The result of the LSAP is the optimal permutation $\boldsymbol{\pi}_2$. The solution is then fixed and stored in the scheduling matrix $\mathbf{S} = [\boldsymbol{\pi}_1^T, \boldsymbol{\pi}_2^T]^T$.
- With the fixed assignments stored in $\mathbf{S}$, the algorithm selects the next adjacent cell c and finds the optimal permutation $\boldsymbol{\pi}_c$ of users with index i_c in slot $k_c(i_c)$ in this cell to the previously fixed assignments based on the 2-dimensional cost matrix, as in (14). The cost matrix applied to this $N > 2$ case is given in (23). The beamforming weights are given by the max–min beamforming problem

$$\boldsymbol{\Omega}_k = \mathrm{argmax}_{\boldsymbol{\Omega}} \min_{i\in\{[\mathbf{S}]_{:,k}\cup i_c\}} \frac{\boldsymbol{\omega}_i^H \mathbf{R}_{i,i}\boldsymbol{\omega}_i}{\sum\limits_{\substack{l\in\{[\mathbf{S}]_{:,k}\cup i_c\}\\ l\neq i}} \boldsymbol{\omega}_l^H \mathbf{R}_{l,i}\boldsymbol{\omega}_l + \sigma_i^2}$$
$$\text{s.t. } |[\boldsymbol{\omega}_i]_a|^2 \leq P_C \;\; \forall a\in\mathcal{A}_i, \quad \forall i\in\{[\mathbf{S}]_{:,k}\cup i_c\}. \tag{25}$$

The result is the optimal 2-dimensional assignment of the users of the new cell c to all previous selected and fixed assignments stored in $\mathbf{S}$. Note, the overall assignment is still suboptimal. However, the result of this dimension-wise optimization is already quite good due to the fact that the majority of the interference is caused by the adjacent cells. To simplify the notation, the index k is removed from the beamforming vectors $\boldsymbol{\omega}_l$. However, the MBF algorithm optimizes in each step k the beamforming

vectors of all selected cells and stores them in a matrix $\mathbf{\Omega}_k$.
- If all N cells are visited, the algorithm terminates and returns the optimal scheduling matrix $\mathbf{S}$ and the set of all optimized beamforming matrices $\mathcal{W}$.

The dimension-wise heuristic optimizes the beamforming vectors only for the already selected cells. In each iteration, the beamforming problem grows by one cell. Especially, at the beginning, the beamforming problems are small. Alg. 2 depicts the details of the implementation.

5) Dimension-wise Max-Min Optimization: Beside the optimization of the sum rate, fairness is often desired. MBF achieves a balanced mean SINR for a given scheduling slot k. However, the mean SINR can vary among different scheduling slots. An optimization of the sum rate is, therefore, not the best strategy if fairness among the users is desired. An optimal solution for a fair assignment is presented in Section III-A2 for the 2-dimensional case. This approach can be extended to the N-dimensional generalized (multidimensional) LBAP:

$$\begin{aligned} \max \quad & \min_{1\le k_1,\dots,k_N\le K} w_{k_1,\dots,k_N} x_{k_1,\dots,k_N} \qquad (26)\\ \text{s.t.} \quad & \sum_{\substack{k_j=1\\ j=1,\dots,N\ j\neq 1}}^{K} x_{k_1,\dots,k_N} = 1, \quad k_1 = 1,\dots,K,\\ & \dots\\ & \sum_{\substack{k_j=1\\ j=1,\dots,N\ j\neq N}}^{K} x_{k_1,\dots,k_N} = 1, \quad k_N = 1,\dots,K,\\ & x_{k_1,\dots,k_N} \in \{0,1\} \quad k_1,\dots,k_N = 1,\dots,K. \end{aligned}$$

In [25], the authors proved the $\mathcal{NP}$-hardness of the 3-index bottleneck assignment problem which follows from the $\mathcal{NP}$-completeness of the 3-dimensional matching problems with cost in $\{0,1\}$. Consequently, the multidimensional linear bottleneck assignment problem is $\mathcal{NP}$-hard as well.

The dimension-wise heuristic Alg. 2 can be simply modified to solve a LABP by using the cost matrix (24). To achieve a max-min fairness over all slots, the dimension-wise heuristic based on a LBAP uses the MBF optimization (24) to balance the mean SINR among users as in Section III-A2 (for the 2-dimensional case). The dimension-wise max–min optimization is similar to the algorithm presented in Section III-B4. The heuristic stores fixed assignments in $\mathbf{S}$ and keeps them unchanged. Then heuristic computes the costs of combinations of the users i_c of a new cell c to the previous assignments based on the resulting mean SINR after MBF. The MBF optimization results in max–min-fair mean SINR for a given slot k among all active users in this slot k. The LBAP, on the other hand, searches for an optimal assignment such that the mean SINR over all scheduling slots k is balanced. Hence, fairness in both directions, among the users and among the slots, can be achieved. Alg. 3 depicts the details of the implementations.

6) Greedy based sum rate maximization: Straightforward heuristics for $\mathcal{NP}$-hard graph problems are based on a greedy strategy. A greedy algorithm always takes the best decision for the moment. It never reconsiders previous decisions, therefore, it is only able to find suboptimal solutions. On the other hand, due to the simple decisions, a greedy algorithm has a low complexity. The greedy approach does not reconsider its previous decisions. Therefore, these decisions should be as good as possible. This approach results in the idea of the developed greedy algorithm.

In a given slot k, the algorithm starts in a randomly selected cell and selects randomly a user. The idea is similar to a puzzle: Rotate the beampattern in an adjacent cell by a proper user selection such that the mutual interference among the selected users is minimized. Hence, the newly found user along with the beam lobe of the BS serving this user perfectly fits to the already selected users. The algorithm uses the following steps.

- For a given slot k, the algorithm starts randomly in a cell of the network and schedules randomly a user i_0.
- The interference, a user receives, is caused almost exclusively from its adjacent BSs. The algorithm continues the search in an adjacent cell c (adjacent to the cells of previous selected users). In the chosen adjacent cell, the greedy algorithm selects the strongest user i_{best} from the set of available unscheduled users.
- The strongest user is the user that maximizes the sum rate achieved in slot k (greedy step).
- After the selection of the strongest user i_{best} in cell c, the algorithm continues the search in the next adjacent cell until all cells are visited. Then, the next slot $k+1$ is optimized in the same way.
- The algorithm terminates until all slots are optimized.

The algorithm requires a set of free users $\mathcal{U}_c$ given for each cell c. According to the QFS criterion, a user with index i can appear several times in the set $\mathcal{U}_c$, therefore, this set is formally a defined as a multiset [30].

Definition 6: Assume the multiset $\mathcal{A}$ contains n_a times the element a: $\mathcal{A} = \{\underbrace{a, a, \dots a}_{n_a \text{ times}}, b, c, \dots\}$. The set minus operation $\mathcal{A} \setminus a$ applied to a multiset $\mathcal{A}$ results in the set $\tilde{\mathcal{A}} = \{\underbrace{a, a, \dots a}_{n_a - 1 \text{ times}}, b, c, \dots\}$.

Definition 7: With Definition 6, the following update function

$$F(\mathcal{A}, \mathcal{B}, a) = \begin{cases} \mathcal{A} \setminus a & \text{if} \quad \mathcal{A} \setminus a \neq \emptyset \\ \mathcal{B} & \text{if} \quad \mathcal{A} \setminus a = \emptyset \end{cases} \qquad (27)$$

for the greedy algorithm can be defined.
The function (27) will remove a selected element (user index) a from multiset (or set of free users) $\mathcal{A}$ if $\mathcal{A}$ without a unequals the empty set. Otherwise, the set is initialized with a new set $\mathcal{B}$. With the definition of the function (27), the greedy algorithm achieves either QFS fairness or ORRS fairness with different initializations of the matrices $\mathcal{U}_c$.

- QFS: The set of free users is a multiset $\mathcal{U}_c$. With $K = \text{LCM}\{n_1, n_2, \dots, n_N\}$, the multiset $\mathcal{U}_c$ contains each user i exactly K/n_c times. Therefore, the BS antenna array of cell c serves each user equally often during the K slots.
- ORRS: The set of free users is a simple set $\mathcal{U}_c$ and contains each user only once. After n_c slots, each user is

Algorithm 4 Greedy based sum rate maximization

Initialize: Compute all $\mathcal{U}_{c,0}$ $\forall c = 1...N$
Set $\mathcal{U}_c = \mathcal{U}_{c,0}$ $\forall c = 1...N$
for $k = 1$ to K **do**
  $\mathcal{C} := \emptyset$
  Randomly take a user i_0 from $\mathcal{U}_{c(i_0)}$
  $[\mathbf{S}]_{c(i_0),k} := i_0$
  Update: $\mathcal{U}_{c(i_0)} := F(\mathcal{U}_{c(i_0)}, \mathcal{U}_{c,0}, i_0)$
  for $c = 1$ to N **do**
    Find next cell c adjacent to the visited cells $\mathcal{C}$
    $\mathcal{C} := \mathcal{C} \cup c$
    for all $i_c \in \mathcal{U}_c$ **do**
      Compute the beamforming matrices $\mathbf{\Omega}_k$ with (25)
      Compute
      $R^{\Sigma}_{i_c} = \sum_{i \in \{[\mathbf{S}]_{:,k} \cup i_c\}} \log(1 + \frac{\boldsymbol{\omega}_i^H \mathbf{R}_{i,i} \boldsymbol{\omega}_i}{\sum_{\substack{l \in \{[\mathbf{S}]_{:,k} \cup i_c\} \\ l \neq i}} \boldsymbol{\omega}_l^H \mathbf{R}_{l,i} \boldsymbol{\omega}_l + \sigma_i^2})$
    end for
    $i_{\text{best}} = \underset{i_c \in \mathcal{U}_c}{\text{argmax}}(R^{\Sigma}_{i_c})$
    $[\mathbf{S}]_{c,k} := i_{\text{best}}$
    Update: $\mathcal{U}_{c(i_{\text{best}})} := F(\mathcal{U}_{c(i_{\text{best}})}, \mathcal{U}_{c,0}, i_{\text{best}})$
  end for
end for
return $\mathbf{S}$, $\mathcal{W}$

scheduled again. Therefore, this initialization results in a ORRS fair scheduling.

The greedy algorithm is depicted in Alg. 4. The index $c(i)$ indicates the cell of user i. At the initialization, the sets (ORRS) or multisets (QFS) $\mathcal{U}_{c,0}$ are initialized for each cell. If ORRS is desired, the set $\mathcal{U}_{c,0}$ contains each user exactly once. If QFS is desired, the set $\mathcal{U}_{c,0}$ contains each user K/n_c times. The algorithm uses working sets or multisets $\mathcal{U}_c = \mathcal{U}_{c,0}$. If a user is scheduled by the greedy approach, the user will be removed from this set with the update function (27). If, e.g., ORRS is desired, the user can not be scheduled until all other users of the set are scheduled. One advantage of the greedy algorithm is, therefore, the possibility of a ORRS fair scheduling. Low delays between two consecutive transmissions are guaranteed. Another advantage is the low complexity.

7) Complexity Analysis: Regarding all beam scheduling algorithms presented in Sections III-B3-III-B6, the computation of the beamforming weights for the cost computation has the largest complexity. The complexity of, e.g., the LSAP to compute the assignments is low compared to the complexity of the beamforming optimization to compute the cost matrix. Therefore, the overall complexity is expressed in terms of required beamforming optimizations. The beamforming optimizations can have different sizes and their complexity depends on the number of participating BS arrays. In what follows, the complexity of a beamforming problem with a size of n cells or BS arrays is denoted by $\mathcal{O}_B(n)$.

Sum Rate based Simulated Annealing: To compute the costs $R(\mathbf{S}_b)$ for the solution based on SA presented in Alg. 1, the beamforming weights for the exchanged columns of the scheduling matrix must be computed. Hence, problem (9) is solved with $\mathbf{S}_b$ as a given scheduling matrix. In total two beamforming problems of size N are solved. Let $\mathcal{O}_B(N)$ be the complexity of a MB problem with N BS arrays the total complexity of the cost computation is given by $2 \cdot \mathcal{O}_B(N)$. Assuming the annealing process needs T_0 iterations the total complexity is

$$C = T_0 \cdot 2 \cdot \mathcal{O}_B(N). \tag{28}$$

Dimension-wise Optimization: The dimension-wise optimizations presented in Sections III-B4 and III-B5 are based on the computation of a cost matrix. The algorithm iterates over all cells with index c. In each iteration, a new cell c is added to the set of the visited cells. Therefore, the beamforming problem grows in each iteration by one BS. Assuming the current cell index is c, then c cells are added to the set of visited cells $\mathcal{C}$. With this assumption, the beamforming algorithm in iteration c has a complexity of $\mathcal{O}_B(c)$, the computation of the cost matrix requires K^2 beamforming optimizations with a complexity of $\mathcal{O}_B(c)$ to compute the cost matrix in iteration c. In total the algorithm visits N cells, therefore, the total complexity is

$$C = K^2 \cdot \mathcal{O}_B(2) + \ldots + K^2 \cdot \mathcal{O}_B(N) = K^2 \cdot \sum_{c=2}^{N} \mathcal{O}_B(c). \tag{29}$$

Sum Rate Based Greedy Algorithm: The greedy algorithm iterates over all slots K and in each slot with index k, the algorithm iterates over all cells. For a given selected cell c, the greedy algorithm searches the strongest user in this cell. Assume each cell has K users, to determine the strongest user, K beamforming optimizations are computed. Hence, the total complexity of slot $k = 1$ is given by

$$C_1 = K \cdot \mathcal{O}_B(2) + \ldots + K \cdot \mathcal{O}_B(N). \tag{30}$$

Then, the algorithm removes the strongest user from the set $\mathcal{U}_c$. In the next step $k = 2$ in cell c the algorithm searches the strongest user out of the set of $K-1$ users and the complexity in step $k = 2$ is

$$C_2 = (K-1) \cdot \mathcal{O}_B(2) + \ldots + (K-1) \cdot \mathcal{O}_B(N)). \tag{31}$$

In the last step just one user is left in each cell and the complexity is simply

$$C_K = \mathcal{O}_B(2) + \ldots + \mathcal{O}_B(N) \tag{32}$$

to compute the last beamforming vectors. The total complexity is then $C = C_1 + \ldots C_K$. Rearranging the sum, the overall complexity is given by

$$\begin{aligned} C = \mathcal{O}_B(2) \cdot (K + (K-1) + \ldots + 1) + \ldots \\ + \mathcal{O}_B(N) \cdot (K + (K-1) + \ldots + 1). \end{aligned} \tag{33}$$

Using the formula of the arithmetic serie, the final complexity can be simplified to

$$C = \frac{(K-1) \cdot K}{2} \cdot \sum_{n=2}^{N} \mathcal{O}_B(n). \tag{34}$$

Comparison: For a large K, the greedy algorithm has approximately half the complexity of the dimension-wise

TABLE II: Simulation parameters

Number of users per user drop and cell	12
Transmit antenna arrays	uniform linear array
Number of antenna array elements at BSs	4
Number of antenna array elements at users	1
Intersite distance	2000 m
Antenna spacing	half wavelength
Path loss exponent	3.76
Power angular density	Laplacian, 10°
Power constraint	per BS array power constraint

optimization approaches in the sense of required beamforming optimizations. The complexity of the SA approach depends on the start temperature T_0. This paper selects a start temperature such that the complexity of the SA approach is as large as the complexity of the two dimension-wise approaches. Consequently, the equality

$$C = K^2 \cdot \sum_{c=2}^{N} \mathcal{O}_B(c) = T_0 \cdot 2 \cdot \mathcal{O}_B(N). \tag{35}$$

must hold. With

$$T_0 = \lfloor \frac{K^2 \cdot \sum_{c=2}^{N} \mathcal{O}_B(c)}{2 \cdot \mathcal{O}_B(N)} \rfloor \tag{36}$$

the complexity (28) of the SA approach is equal to the complexity of the dimension-wise approaches.

IV. Results and Discussion

A. 2-Cell Scenario

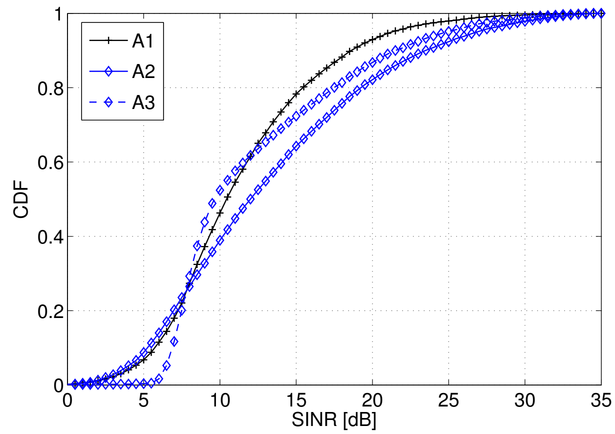
The results presented in this section are based on the 2-cell scenario which corresponds to two adjacent cells of the 21-cell scenario depicted in Figure 1. Tab. II summarizes the main simulation parameters. As in [31], in each user drop, the users are uniformly distributed in the network. Based on the location of users relative to the BS arrays (uniform linear), the spatial correlation matrices are calculated with the assumption of a Laplacian power angular density distribution. This is a common assumption in outdoor scenarios [32].

Fig. 5(a) depicts the cumulative distribution functions (CDFs) of the following algorithms:

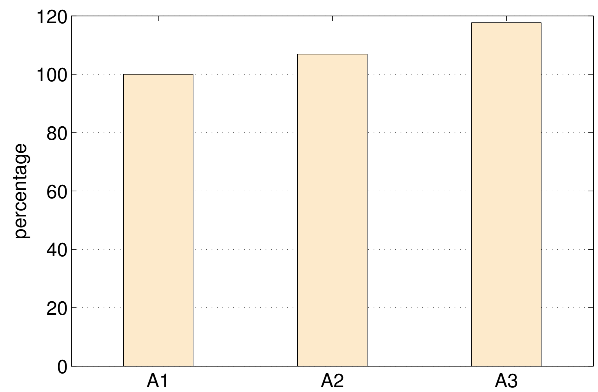
- A1: MBF and random scheduling
- A2: MBF and optimized scheduling based on a LSAP presented in Section III-A1
- A3: MBF and optimized scheduling based on a LBAP presented in Section III-A2

As expected, algorithm A2, where a maximized sum rate is desired, maximizes the overall sum rate with a marginal impairment of the fairness. The fair LBAP improves the fairness and is able to outperform the random solution also for higher SINRs.

Figure 5(b) depicts the sum rate (8) gains of the algorithms A2 and A3 relative to A1. As expected, A2 has the best sum rate performance with nearly 20% performance gain. However, even the max–min fair approach A3 can achieve an improvement of the sum rate.



(a) CDF of the mean SINR.



(b) Sum rate (8) gains relative to A1 in percentage.

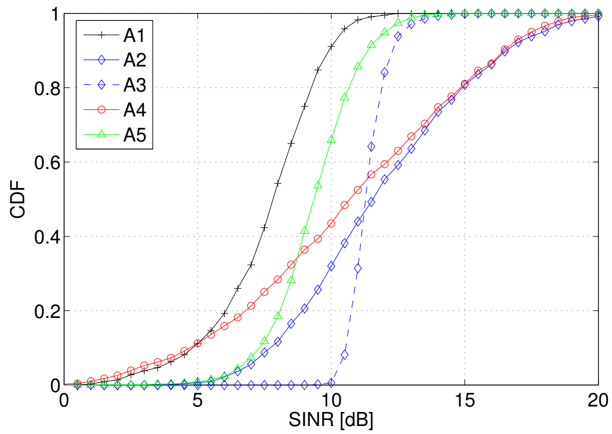
Fig. 5: Performance (SINR and sum rate) of the 2-cell scenario.

These gains are marginal. The reason for these small gains is the limited scenario. Only two cells are regarded. Consequently, the degrees of freedom for a scheduling optimization are limited compared to a scenario with a large number of cells. Therefore, the following section presents an optimization scenario with $N = 21$ cells. In this case, the beam scheduling problem is $\mathcal{NP}$-hard, however, the due to the large number of cells, more degrees of freedom for an optimized beam scheduling are available.

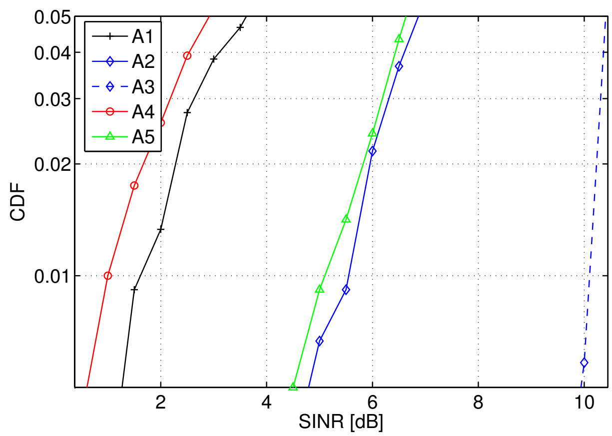
B. $N > 2$-Cell Scenario

The simulation parameters are the same as in Tab. II. However, in this section $N = 21$ cells are optimized. For a fair comparison of the SINR performance, all cells have the same number of active users, therefore, all algorithms satisfy the QFS constraint according to Definition 5. The CDFs of the following algorithms are compared:

- A1: MBF and random scheduling
- A2: Dimension-wise sum rate maximization according to Section III-B4
- A3: Dimension-wise max–min SINR optimization according to Section III-B5
- A4: Greedy sum rate maximization according to Section III-B6

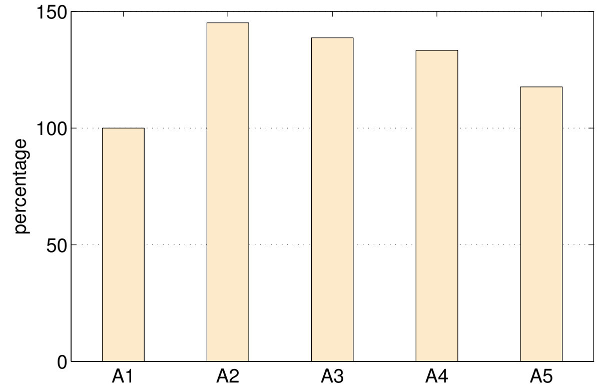


(a) CDF of the mean SINR.

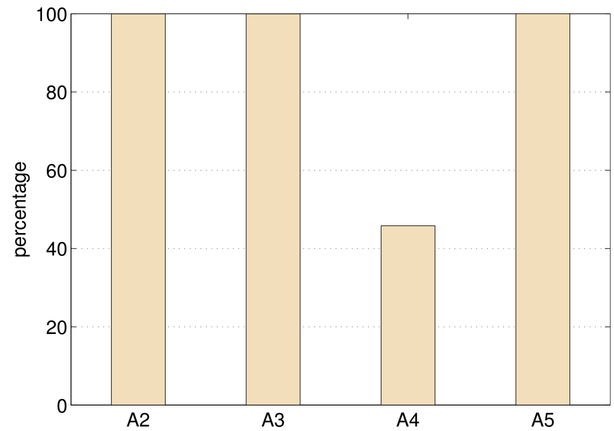


(b) CDF of the mean SINR of the lowest 5%.

Fig. 6: Comparison of the SINR CDF of the different algorithms A1-A5.



(a) Sum rate (8) gains relative to A1 in percentage.



(b) Relative complexity percentage (A2, A3 and A5 correspond to 100%).

Fig. 7: Comparison of sum rate performance and complexity of the new algorithms.

- A5: SA based sum rate maximization according to Section III-B3

Figure 6(a) depicts the CDF of the mean SINR of all algorithms A1-A5. As expected all algorithms outperform the random ORRS (A1) concerning the performance of high SINR values. The greedy algorithm (A4) is slightly weaker for the low SINR region but achieves good results for a high SINR. Regarding the high SINR values, the dimension-wise sum rate maximization (A2) achieves the best result. The fair version of a dimension-wise optimization (A3) is the best approach in terms of the SINR fairness. However, as expected, the strongest users do not gain as much from this approach. The simple SA based approach (A5) is outperformed by all approaches for the strongest 35% of the users. Only for the weakest users, the simlated annealing approach achieves better results than A4.

The new solution A3 has the advantage of an improved SINR fairness compared to the sum rate based solution A2. The weakest 45% of the users can gain compared to A5 and compared to the random solution A1 all users can gain.

An often used indicator in multicell optimization is the performance of the weakest 5% of the users. Figure 6(b) shows the mean SINR of the weakest 5% of the users in all slots and simulation runs. Regarding fairness, the max–min fair dimension-wise approach A3 achieves a significant performance gain. Hence, this approach can be useful in applications where fairness among the users is desired. The greedy approach (A4) is very agressive in maximizing the sum rate. In each slot, the best users are selected cell-by-cell. Consequently, in the last slots only weak users are left which result in low SINR values.

As it can be observed from Fig. 7(a), all proposed algorithms outperform the random and SA approach in terms of the sum rate performance. The new max–min fair scheduling algorithm A3 achieves the best SINR fairness and a system with the sum rate based dimension-wise approach will have the best sum rate performace. Figure 7(b) depicts the relative complexity of all algorithms. According to [33], the complexity of the max–min fair beamforming approach grows cubically in the number of users N. Hence, the complexity of each beamforming problem of size N is in the order of $\mathcal{O}_B(N) = \mathcal{O}(N^3 \cdot K_C)$. Here, K_C denotes a constant factor. Regarding the trade-off between complexity and sum rate performance, the greedy approach A4 is the best approach. To have a better comparison of the results, the initial temperature

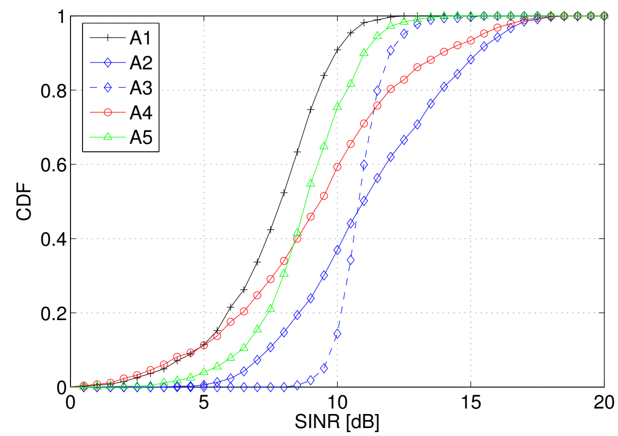
TABLE III: Summary of the advantages/disadvantages of the different algorithms regarding different properties ((++): very good, (+): good, (0): fair, (-): weak).

Algorithm	Temporal fairness	Sum rate	SINR fairness	Complexity
A1: random RRS	(++)	(-)	(-)	(+)
A2: dim.-wise sum rate maximization	(0)	(++)	(+)	(-)
A3: dim.-wise max–min optimization	(0)	(+)	(++)	(-)
A4: greedy sum rate maximization	(+)	(+)	(-)	(0)
A5: SA based sum rate maximization	(0)	(0)	(0)	(-)

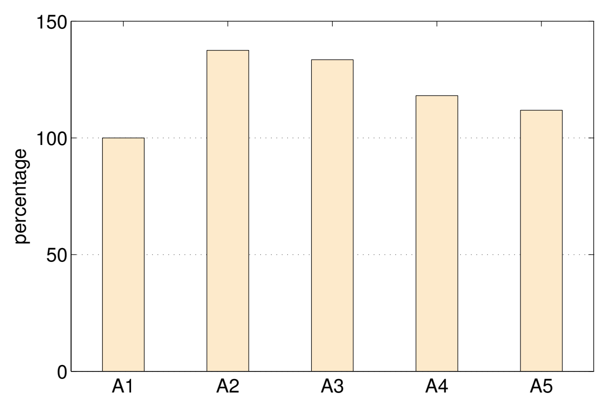
of the SA algorithm A5 is chosen so that the order of the complexity of the SA is equivalent to the dimension-wise approaches A2 and A3. For a large number of users per cell K, the greedy algorithm A4 requires the half number of beamforming optimizations compared to A2, A3, and A5. Additionally, A4 can guarantee the ORRS fairness constraint. Therefore, this approach may be useful in systems where the delay among two consecutive transmission to a users can not be too large. The other approaches satisfy the QFS constraint[1]. By definition of the algorithms, all scheduling heuristics are guaranteed to converge to a local optimal solution. The SA heuristic termitates if the temperature reaches zero (see Alg. 1) and the dimension-wise optimization based algorithms (Alg. 2 and 3) terminate when all cells are visited. However, due to the $\mathcal{NP}$-hard optimization problem, the solutions found by the heuristics, are not guaranteed to be globally optimal. Table III summarizes the results of the algorithms A1-A5 regarding temporal fairness, sum rate performance, SINR fairness, and complexity.

C. Different Numbers of Users per Cell

Fig. 8 shows the results for the case of different numbers of users per cell. For this simulation, the cells contain 4, 6, and 12 users respectively. Due to the reduced multi-user diversity all algorithms have a reduced performance compared to the case where each cell contains 12 users (Fig. 6(a) and Fig. 7(a)). The number of slots is given by $\mathrm{LCM}\{4, 6, 12\} = 12$. The different number of users per cell may result in different delays. The algorithms A2, A3, A6 satisfy the QFS constraint. The greedy algorithm A5 is able to satisfy the ORRS constraint. However, this stricter temporal fairness constraint causes a small performance loss compared to the case of an equal number of users per cell. Therefore, the greedy approach loses more performance compared to the other algorithms in the case of a different number of users per cell.



(a) CDF of the mean SINR.



(b) Sum rate (8) gains relative to A1 in percentage.

Fig. 8: Mean SINR and sum rate performance of the new algorithms for different numbers of users per cell.

V. Summary

This paper presents the general graph theoretic background of the multicell beam scheduling problem. It proves the $\mathcal{NP}$-hardness of the general N-cell beam scheduling problem and presents three useful heuristics which achieve a higher balanced SINR in a multicell scenario. A maximized sum rate or optimized SINR fairness can be achieved. Additionally, in the 2-cell scenario, optimal algorithms concerning fairness and sum rate are presented. The proposed approaches jointly optimize the beamforming vectors, multiuser scheduling, and control the transmit power. Numerical results for the general N-cell case show a large SINR performance gain for the weakest users if the max–min fair approach used.

[1]To achieve a lower delay a post-processing for the scheduling matrix can reduce the delays. This can be done by rearranging the columns of the scheduling matrix, without any effect on the SINR.

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Erratum: Application of Graph Theory to the Multicell Beam Scheduling Problem

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- The power angular density in Table II was wrong. The value of the Laplacian distribution is $10°$.
- The Figures for the sum rate Fig. 5b, Fig. 7a, and Fig. 8b are wrongly calculated. Difference to the IEEE version are marginal, however, to be correct we inserted the corret plots in this version.