On the Pareto Optimum of Long-Term Max-Min Beamforming with General Power Constraints

Guido Dartmann, Xitao Gong, and Gerd Ascheid
Chair for Integrated Signal Processing Systems, RWTH Aachen University
Email: {dartmann, gong, ascheid}@iss.rwth-aachen.de

***Abstract*—In this paper, the Pareto optimum of the max-min beamforming problem with general power constraints is discussed. Conditions without a balanced signal to interference plus noise ratio are identified and based on the discussions an improved direct and iterative solution of the max-min beamforming problem is proposed. Numerical results show the performance gain of the presented solution compared to a classical bisection based approach. The presented solution is able to jointly find the optimized beamforming weights and an assignment of base stations (BSs) to users based on the optimization of the beamformers if a cooperative multipoint transmission is feasible. In the case of long-term channel state information at the BSs, a diversity gain can be achieved.**

I. Introduction

In future wireless networks, the cooperative multipoint (CoMP) transmission could be a promising technology to improve the signal to interference and noise ratio (SINR) of, e.g., cell edge users in single frequency networks.

Scenario: This paper investigates a multicell network as shown in Fig. 1. Each of these small cells has three cooperative base stations (BSs). Every BS has N_A antennas and each user has a single antenna. For simplification of the notations only one user per cell is active and served by the three BSs simultaneously. It is further assumed that all BSs in the network get the long-term channel state information (CSI) of all other jointly active links in the network.

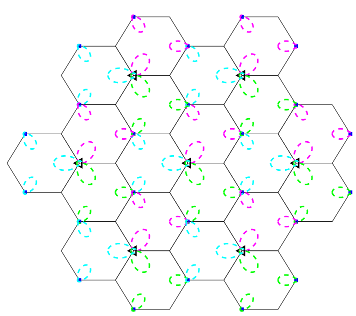

Fig. 1: Simulation scenario: The lobes show the orientation of the antenna pattern. Users inside a cell are served by three BSs.

Related work: The work of Yu et al. [1] is one of the first discussions of the transmit beamforming problem with general power constraints. In their paper, the power minimization problem (PMP) with per-antenna power constraints is discussed. The PMP finds a solution for the beamforming vectors with a minimized total transmit power for a given set of SINR constraints. The authors in [1] prove the convexity of the PMP based on instantaneous CSI and with general power constraints. Additionally, they propose a direct iterative solution with less complexity compared to standard convex solver based approaches. In contrast to the PMP, the max-min beamforming problem (MBP) is able to find the minimal balanced SINR so that no power constraint is violated. This problem is more general and quasi-convex for instantaneous CSI [2]. The authors in [2] propose a solution based on a bisection over a set of convex feasibility check problems to solve the MBP with instantaneous CSI and with general power constraints. This bisection can iterate arbitrarily tight to the global optimum if the feasibility check problems are convex [3]. In [4], the connection among the PMP and the MBP is discussed. The PMP can be also seen as a feasibility check problem of the MBP. The authors propose a bisection over the quality of service SINR of the PMP to solve the MBP. Beside this solution, the recent work [5] proposes a direct solution of the MBP based on the Lagrangian dual problem of the MBP. The iterative solution in [5] is able to find the same solution like the convex solver based bisection method in [2] with less complexity if a balanced SINR for a given set of per-antenna power constraints exists.

Contributions: This paper is mainly an extension of the previous works [5] and [4]:

- In [2], [5], an iterative and direct solution for the MBP in the case of a coherent CoMP transmission based on instantaneous CSI is proposed. The presented solution achieves the same balanced SINR like the classical bisection based solution with a convex solver. A beamforming gain can be achieved if the BSs are perfectly synchronized. In this paper, the MBP is optimized based on long-term CSI. Due to the max-min fairness, a gain in the spatial domain can be achieved. Only BSs close to the users are active, the others are switched of to avoid intercell interference. The presented solution is able to out-perform the convex solver based bisection method for a given accuracy of $\epsilon = 10^{-6}$.
- The previous presented solutions, like in [5], always assume the existence of a balanced SINR for a given set of power constraints. In this paper, conditions where such a balanced SINR for a given set of power constraints does not exist, are identified.
- Based on the considerations concerning the unbalanced SINR conditions, the solution in [5] is extended to a scenario without a balanced SINR for the given set of power

constraints. Numerical results show the performance of the presented solution compared to the convex solver based solution like in [2].

Notation: Lower case and upper case boldface symbols denote vectors and matrices respectively. The nth element of a vector is denoted with $[\mathbf{a}]_n$. The element with indices n, m of a matrix $\mathbf{A}$ is denoted with $[\mathbf{A}]_{n,m}$. The transpose conjugate of a matrix $\mathbf{A}$ is denoted with $\mathbf{A}^H$. The operator $\mathrm{vec}(\mathbf{A})$ denotes the vector operation applied to matrix $\mathbf{A}$. The cardinality of a set $\mathcal{S}$ is given by $|\mathcal{S}|$. The conjugate is denoted with $()^*$. The notation $\oplus_{i=1}^{N}\mathbf{A}_i = \mathrm{diag}(\mathbf{A}_1, \ldots, \mathbf{A}_N)$ denotes the direct sum of the matrices $\mathbf{A}_i$.

II. System setup and data model

In this paper, a network with N_C cooperating BSs is investigated. A user inside a cell is served by three stations each equipped with N_A antennas, as depicted in Fig. 1. At a time instance in each cell c one user i is jointly served by the three BS belonging to this cell c. Let $\mathcal{B}_i$ be the set of cooperating BSs for user i, then the signal r_i user i at a time instant is

$$r_i = \sum_{c\in\mathcal{B}_i} \hat{\mathbf{h}}_{i,c}^H \hat{\mathbf{w}}_c s_i + \sum_{k\in\bar{\mathcal{B}}_i} \hat{\mathbf{h}}_{i,k}^H \hat{\mathbf{w}}_k s_k + n_i, \qquad (1)$$

where $\hat{\mathbf{h}}_{i,c} \in \mathbb{C}^{N_A\times 1}$ is the channel vector from the cth BS to the ith user. The set $\bar{\mathcal{B}}_i$ denotes the set of interfering BSs such that $\mathcal{B}_i \cap \bar{\mathcal{B}}_i = \emptyset$ and $\mathcal{B}_i \cup \bar{\mathcal{B}}_i = \mathcal{C}$, $\mathcal{C}$ being the set of the currently active BSs. $\hat{\mathbf{w}}_k \in \mathbb{C}^{N_A\times 1}$ is the transmit beamforming vectors at BS k, s_i is the information signal to user i with $E\{|s_i|^2\} = 1$ and $E\{s_k s_i^*\} = 0$ if $i \neq k$. The noise signal plus the interference of other networks is given by n_i. To simplify the notation, the channel vectors of the cooperating BSs are given by set $\mathcal{B}_l$ can be stacked into a large virtual antenna array $\mathbf{h}_{l,i}$. This corresponds to a channel vector $\mathbf{h}_{l,i}$ between the virtual array serving user l to user i. The same can be done with the precoding vectors at the BSs of the set $\mathcal{B}_l$, which results in a large virtual beamforming vector $\boldsymbol{\omega}_l$. Using these notations the received signal can be rewritten as:

$$r_i = \mathbf{h}_{i,i}^H \boldsymbol{\omega}_i s_i + \sum_{l\in\mathcal{S}, l\neq i} \mathbf{h}_{l,i}^H \boldsymbol{\omega}_l s_l + n_i. \qquad (2)$$

Here $\mathcal{S}$ denotes the set of indices of users or cells with one scheduled user. The perfect knowledge of instantaneous CSI and a perfect synchronization among the cooperative BSs is very challenging in large networks. Instead of the instantaneous CSI, a more practically relevant approach is the usage of the long-term CSI because of its long stationarity compared to the instantaneous CSI [6]. The assumption of long-term CSI results in the mean SINR where an additional averaging over the channel realizations $\mathcal{H}$ is done. With the assumption $E\{|n_i|^2\} = \sigma_i^2$, the mean SINR is defined by:

$$\gamma_i^D = \frac{E\{|\mathbf{h}_{i,i}^H \boldsymbol{\omega}_i|^2\}}{\sum_{\substack{l\in\mathcal{S}\\ l\neq i}} E\{|\mathbf{h}_{l,i}^H \boldsymbol{\omega}_l|^2\} + \sigma_i^2} = \frac{\boldsymbol{\omega}_i^H \mathbf{R}_{i,i} \boldsymbol{\omega}_i}{\sum_{\substack{l\in\mathcal{S}\\ l\neq i}} \boldsymbol{\omega}_l^H \mathbf{R}_{l,i} \boldsymbol{\omega}_l + 1}. \qquad (3)$$

Assuming the channels of the different links are uncorrelated, the spatial correlation matrices are given by:

$$\mathbf{R}_{l,i} = \frac{1}{\sigma_i^2} E\{\hat{\mathbf{h}}_{l,i} \hat{\mathbf{h}}_{l,i}^H\} = \frac{1}{\sigma_i^2} \oplus_{c\in\mathcal{B}_l} E\{\hat{\mathbf{h}}_{i,c} \hat{\mathbf{h}}_{i,c}^H\}. \qquad (4)$$

Beside the downlink (DL) SINR (3), the following definition of the uplink (UL) SINR is used. The UL (receive) beamforming vectors of a single BS are given by $\hat{\mathbf{v}}_c$. Concatenating the receive beamforming vectors $\hat{\mathbf{v}}_c$ of the set of BSs ($c \in \mathcal{B}_i$) from a user i in a large vector $\mathbf{v}_i$ in the UL, and with $\mu_{i,a} \in \mathbb{R}^+ \ \forall a \in \mathcal{A}_i$, where $\mathcal{A}_i$ is the set of array elements serving user i and $\mathcal{A} = \bigcup_{i\in\mathcal{S}} \mathcal{A}_i$ (with $N_T = |\mathcal{A}|$) denotes the total set of antenna elements, and the definition of the matrix $\mathbf{M}_i = \oplus_{a\in\mathcal{A}_i} \mu_{i,a}$, the dual UL SINR of the virtual BS array serving user i is given by:

$$\gamma_i^U = \frac{\lambda_i \mathbf{v}_i^H \mathbf{R}_{i,i} \mathbf{v}_i}{\mathbf{v}_i^H (\mathbf{M}_i + \sum_{\substack{l\in\mathcal{S}\\ l\neq i}} \lambda_l \mathbf{R}_{i,l}) \mathbf{v}_i}. \qquad (5)$$

III. Optimization Problem and the Uplink Downlink Duality

It is desired to improve the worst SINR of the currently scheduled users with the power $p_a = |[\boldsymbol{\omega}_i]_a|^2$ at each antenna element constrained by $P_A, \forall a \in \mathcal{A}$. The MBP can be stated as

$$\max_{\boldsymbol{\Omega}} \min_{i\in\mathcal{S}} \quad \gamma_i^D \qquad (6)$$

$$|[\hat{\boldsymbol{\omega}}]_a|^2 \leq P_A \quad \forall a \in \mathcal{A}.$$

Here, the matrix of all beamformers is $\boldsymbol{\Omega} = [\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_M]$, and $\hat{\boldsymbol{\omega}} = \mathrm{vec}(\boldsymbol{\Omega})$. For arbitrary matrices $\mathbf{R}_{l,i}$ the MBP is non-convex. For rank-1 matrices $\mathbf{R}_{l,i} = \mathbf{r}_{l,i}\mathbf{r}_{l,i}^H$ the MBP can be proven to be quasi-convex [2], [5]. With a sum power constraint this problem can be optimally solved for long-term CSI as well [7]. The MBP can be solved with the so called dual UL problem with less complexity. In [5], the dual UL problem is derived base on the Lagrangian dual problem. The algorithm proposed in [5] is based on the duality of the inner problem with a fixed $\boldsymbol{\mu}$. The dual UL problem can be separated in an outer and in inner problem [5]. With the definitions of the matrix $\mathbf{V} = [\mathbf{v}_1, \ldots, \mathbf{v}_M]$ and the vectors $\boldsymbol{\lambda} = [\lambda_1, \ldots, \lambda_M]$ and $\boldsymbol{\mu}$ composed of all $\mu_{i,a}$s, $\forall i \in \mathcal{S}$, $\forall a \in \mathcal{A}_i$, the inner problem of the dual UL problem of the MBP (6) with per-antenna power constraints is given by:

$$f^U(\boldsymbol{\mu}) = \max_{\boldsymbol{\lambda},\mathbf{V}} \min_{i\in\mathcal{S}} \gamma_i^U \qquad (7)$$

$$\text{s.t.} \quad \boldsymbol{\lambda}^T \cdot \mathbf{1} \leq P \quad \lambda_i \geq 0, \ \ \forall\, i \in \mathcal{S}, \qquad (8)$$

where the sum power constraint $P = \boldsymbol{\mu}^T\mathbf{1} \cdot P_A$ for the weighted sum power is constant for fixed $\boldsymbol{\mu}$. The corresponding DL MBP where the weighted sum power is limited to P is given by:

$$f^D(\boldsymbol{\mu}) = \max_{\boldsymbol{\Omega}} \min_{i\in\mathcal{S}} \gamma_i^D \qquad (9)$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{S}} \boldsymbol{\omega}_i^H \mathbf{M}_i \boldsymbol{\omega}_i \leq P, \qquad (10)$$

where $\mathbf{M}_i \succeq 0$. For a fixed $\boldsymbol{\mu}$, this problem is a MBP with a weighted sum power constraint. The problem (7), (8) is the Lagrangian dual problem of problem (9), (10). The proof is a straightforward extension of the proof presented in [8]. The algorithm presented in Section V is based on this duality and finds proper $\mu_{i,a}$s so that the per antenna power constraints are fulfilled if a balanced SINR exists.

IV. Pareto Optimality

In this chapter the theoretical background concerning the optimality of the MBP is investigated. In [5], [9], the assumption of a balanced SINR for a given noise level and a given set of power constraints is used. Hence, it is assumed there exist a Pareto-optimal solution [10] of the DL SINR:

Definition 1: A tuple $(\gamma_1^D, \gamma_2^D, \dots, \gamma_M^D)$ is Pareto optimal if there is no other tuple $(\hat{\gamma}_1^D, \hat{\gamma}_2^D, \dots, \hat{\gamma}_M^D)$ with

$$(\hat{\gamma}_1^D, \hat{\gamma}_2^D, \dots, \hat{\gamma}_M^D) \geq (\gamma_1^D, \gamma_2^D, \dots, \gamma_M^D)$$

(the inequality is component-wise) and

$$(\hat{\gamma}_1^D, \hat{\gamma}_2^D, \dots, \hat{\gamma}_M^D) \neq (\gamma_1^D, \gamma_2^D, \dots, \gamma_M^D)$$

Definition 2: Networks, where a Pareto optimal balanced DL SINR for the given power constraints exists, are defined as balanced interference coupled networks.

This paper investigates a more general case, where for a decoupled structure of the network for a given set of per antenna power constraints or per stations power constraints, a balanced SINR is not feasible. The results of this discussion are used to improve the algorithm in [5]. This section considers a per cell power constraint P_C to simplify the discussion. The SINR of a user $i \in \mathcal{S}$ can be expressed by:

$$\gamma_i^D = \frac{p_i}{\sum_{\substack{l \in \mathcal{S} \\ l \neq i}} p_l G_{l,i} + \nu_i}, \qquad (11)$$

with the resulting interference attenuation and an effective noise level respectively:

$$G_{l,i} = \frac{\mathbf{v}_l^H \mathbf{R}_{l,i} \mathbf{v}_l}{\mathbf{v}_i^H \mathbf{R}_{i,i} \mathbf{v}_i}, \text{ and } \nu_i = \frac{1}{\mathbf{v}_i^H \mathbf{R}_{i,i} \mathbf{v}_i}. \qquad (12)$$

The optimality conditions for more general power constraints need to be discussed at first. The optimality condition for a per cell constrained max-min optimum is formulated in the following well known proposition.

Proposition 1: If $\gamma^* = \max_{\mathbf{p}} \min_{k \in \mathcal{S}} \gamma_k^D$ and subject to $p_i \leq P_C$, then at least one $p_k \in \mathbf{p}$ will fullfill $p_k = P_C$.

Proof: Contrary: Assuming all $p_k < P_C$ and it is further assumed that:

$$\gamma_k^D = \frac{p_k}{\sum_{\substack{l \in \mathcal{S} \\ l \neq k}} p_l G_{l,k} + \nu_k} = \gamma^*, \qquad (13)$$

then there exist a $\delta > 1$ that at least one $\delta \cdot p_i = P_C$ so that

$$\gamma_k^D = \frac{\delta \cdot p_k}{\sum_{\substack{l \in \mathcal{S} \\ l \neq k}} \delta p_l G_{l,k} + \nu_k} = \frac{p_k}{\sum_{\substack{l \in \mathcal{S} \\ l \neq k}} p_l G_{l,k} + \nu_k / \delta} > \gamma^* \qquad (14)$$

holds. ■

Obviously, users served by BSs transmitting with the full power P_C are the bottleneck.

Coupled Network: At first, the case of a network with a balanced SINR is discussed.

Definition 3: A user k is coupled with the network if all $G_{l,k} > 0$ and $G_{k,l} > 0 \; \forall l \neq k$.

Assuming, there exist one weakest user k with

$$I_k = \sum_{\substack{l \in \mathcal{S} \\ l \neq k}} p_l G_{l,k} + \nu_k \qquad (15)$$

and $I_k = \max(I_1, \dots I_M)$. Then, the BS serving this user k will transmit with the full power $p_k = P_C$ according to Proposition 1. The resulting SINR of user k is then given by:

$$\gamma_k^D = \frac{P_C}{\sum_{\substack{l \in \mathcal{S} \\ l \neq k}} p_l G_{l,k} + \nu_k}.$$

The SINR is balanced if the network is coupled:

$$\gamma^* = \gamma_1^D = \dots = \gamma_k^D = \dots = \gamma_M^D$$

By increasing the noise level ν_k an unbalanced SINR could be created. Assuming the noise level ν_k will be scaled by with a $\delta_k > 1$ so that

$$\gamma_k^D = \frac{P_C}{\sum_{\substack{l \in \mathcal{S} \\ l \neq k}} p_l G_{l,k} + \nu_k} > \frac{P_C}{\sum_{\substack{l \in \mathcal{S} \\ l \neq k}} p_l G_{l,k} + \delta_k \cdot \nu_k}. \qquad (16)$$

The SINR of user k is now decreased without any effect of the SINR to the other users $l \neq k$, their SINR is still balanced. The power of BS k can not be increased to recover a balanced SINR, because it reaches already its power constraint. But if the network is coupled the $G_{l,k} > 0 \; \forall l \neq k$, each BS $l \neq k$ can reduce its own power to balance the SINR of all users again.

Decoupled Network: A special structure of the network can result in conditions, where a balanced SINR does not exist for a given set of power constraints. In this paper two cases are identified. The SINR remains unbalanced if a user is physically decoupled from the network, e.g., the user could be strongly protected from intercell interference by buildings, strong antenna patterns or zero forcing beamforming. From the first impression this could be a good condition for those users, on the other hand the algorithms requiring the existence of a balanced SINR can not converge to a feasible solution in this case. At first, a formal definition for the decoupled user is needed.

Definition 4: A user i is called decoupled from the network with per cell power constraints,

1) if this user does not receive interference from other users $j \neq i$, thus, $G_{j,i} = 0 \; \forall j \neq i$,
2) or if this user does not generate any interference to other users $j \neq i$, thus, $G_{i,j} = 0 \; \forall j \neq i$.

Proposition 2: For a given set of power constraints P_C it is not always possible to find a balanced SINR γ^* if there exist a user k which is decoupled from the network.

Proof: In the two cases of Definition 4, a balanced SINR can not be recovered:

1) Assuming user k is the weakest user in the network, which can be achieved by an arbitrary large δ_k like in (16) and there is no intercell interference every $G_{l,k} = 0$ $\forall l \neq k$, then the SINR of user k is given by:

$$\gamma_k^D = \frac{P_C}{\delta_k \cdot \nu_k} \qquad (17)$$

Now, there is no possibility by power control to recover a balanced SINR. The SINR of user k can be made arbitrary small by increasing δ_k and the BS k can only transmit with the maximal power $p_k = P_C$, because it can not exceed the power constraint P_C. The other other BSs can not influence the SINR of user k.

2) Another situation is the opposite case, where the user k is not the weakest user and its BS does not create interference to other users $G_{k,l} = 0 \; \forall l \neq k$. Then the transmitted power of BS k to user k can not reduce the SINR of the other users l. The BS k can increase its

transmit power until $p_k = P_C$. The resulting SINR γ_k^D can be larger than the SINR γ_l^D of the other users $l \neq k$, because user k does not create any interference to the residual network.

■

V. Algorithm

The MBP with general power constraints can be solved with a bisection over a convex program of the MBP (6) with a fixed SINR constraint γ like in [11] if instantaneous CSI is used, or like in [5] if long-term CSI is used. Beside the solution based on a convex solver, direct solutions are proposed in [5], [9], [4]. In this section, the solution proposed in [5] is extended to scenarios without a balanced SINR as presented in Proposition 2. For a fixed $\boldsymbol{\mu}$, the inner problem (9) can be solved based on the duality to (7) like in [7] or with a reduced complexity like in [8], [5]. The solution described in [5] is based on a simple vector iteration and converges rapidly. The inner problem corresponds to a weighted sum power constrained MBP. Remember, the MBP with general power constraints is non-convex in general or at least quasi-convex in the case of instantaneous CSI [2], therefore, a global optimal solution might not be found in every scenario. In [5], [9], the per antenna or per BS station power constraints are fulfilled by an outer optimization over the variable $\boldsymbol{\mu}$. Simulation results have shown, if a balanced SINR exist, the algorithm converges to the same solution like the bisection algorithm based on a convex solver [5]. In [5], the $\mu_{i,a}$s are computed based on the beamforming vectors of the inner function (9). Comparing constraint (10) of this function with $P = \boldsymbol{\mu}^T \mathbf{1} \cdot P_A$ of the outer maximization, both are weighted sums over $\boldsymbol{\mu}$. Since at the optimum the constraint is satisfied with equality

$$\sum_{i \in \mathcal{S}} \boldsymbol{\omega}_i^H \mathbf{M}_i \boldsymbol{\omega}_i = \boldsymbol{\mu}^T \mathbf{1} \cdot P_A, \tag{18}$$

the $\mu_{i,a}$s can be updated by comparing the power coupled with each $\mu_{i,a}$ in (10) with P_A and then scaling the $\mu_{i,a}$s such that the constraint in (18) is satisfied with equality. This idea results in the following update:

$$\tilde{\mathbf{M}}_i = \frac{\operatorname{diag}(\boldsymbol{\omega}_i \boldsymbol{\omega}_i^H)\hat{\mathbf{M}}_i}{P_A}, \quad \forall i \in \mathcal{S} \tag{19}$$

$$\mathbf{M}_i = \zeta \tilde{\mathbf{M}}_i \ \text{ with } \ \zeta = \frac{\mathbf{1}^T \mathbf{p}}{\sum_{i \in \mathcal{S}} \operatorname{Tr}\{\tilde{\mathbf{M}}_i\} P_A}.$$

Here, $\operatorname{diag}(\mathbf{A})$ is a diagonal matrix with the elements of the main diagonal of $\mathbf{A}$ and $\hat{\mathbf{M}}_i$ denotes the matrix of the previous iteration. The matrices $\mathbf{M}_i$ correspond to the noise levels in the UL SINRs (5). If a matrix $\mathbf{M}_i$ has large entries on its main diagonal, the noise level in the corresponding uplink is large, which results in a large DL power after solving the DL problem (9) to achieve the same balanced SINR. This is because of the duality. On the other hand results a matrix $\mathbf{M}_i$ with small entries on its main diagonal in a smaller DL power. With the update (19), the entries on the main diagonal of matrix $\mathbf{M}_i$ become larger than one if the power constraint is violated and they become smaller than one if it is not violated. The noise in the UL SINR (5) will be accordingly scaled.

In interference decoupled scenarios, the power constraints can be violated, like in (17), because the algorithm assumes always a balanced SINR [5]. For example, consider the case 1) in the proof of Proposition 2. Assuming there is a weakest user k with a SINR like in (17). The algorithm of [5], with the assumption of a balanced SINR will increase its power above the power constraint P_A to achieve a balanced SINR. Then it can happen that the entries of the main diagonal of matrix $\mathbf{M}_i$ become to large. To prevent this case the following extension of the update (19) is made if the power constraint is violated:

$$[\hat{\boldsymbol{\omega}}]_a = [\hat{\boldsymbol{\omega}}]_a \frac{\sqrt{P_A}}{|[\hat{\boldsymbol{\omega}}]_a|}. \tag{20}$$

Another approach to avoid a decoupled network could be the avoidance of scheduling decisions where a users are decoupled from the network. The final outer loop is listed in Alg. 1.

TABLE I: Simulation parameters

Number of user drops	1000
Transmit antenna arrays	ULA
Number of array elements at BS	4
Number of array elements at MS	1
Interference of adjacent networks	By ring of omnidir. BSs
Antenna spacing	half wavelength
Shadowing standard deviation	5dB
Path loss exponent	3.76
Power angular density	Laplacian, 33°
Power constraint	per-antenna

Algorithm 1 Outer loop: DL Power and iterations over $\boldsymbol{\mu}$

Initialize $\boldsymbol{\mu} = \mathbf{1}$
repeat
  Inner loop like in [5] $\rightarrow \mathbf{V}$ and γ^D
  Power control like in [5] $\rightarrow \mathbf{p}$
  Update $\boldsymbol{\Omega}$ according to (20)
  Update $\boldsymbol{\mu}$ according to (19)
until Convergence
return $\boldsymbol{\Omega}$

VI. Simulation results

The simulation scenario corresponds to the scenario depicted in Fig. 1. Further simulation parameters are summarized in Tab. I. Three algorithms are investigated in this section:

- A1: The iterative algorithm [5] with outer and inner loop optimization and with the additional update (20) in the outer loop.
- A2: The iterative algorithm [5] without the the additional update (20) in the outer loop.
- A3: The bisection method like in [2] for instantaneous CSI or like in [5] for long-term CSI.

All algorithms use an accuracy of $\epsilon = 10^{-6}$. Three BSs can serve a user in a cell. The proposed architecture with two additional BSs in the cell edge region results in a higher gain for all users compared to the single BS per cell case (see Fig. 2). The extended algorithm in this paper finds the same SINR like the convex solver based bisection method. Fig. 3 depicts the SINR as a function of the location. The color denotes the SINR. High SINR corresponds to red areas, low SINR corresponds to blue areas. Only BSs close to users are active and these BSs focus their power in the direction of the served users, to avoid intercell interference. To achieve an

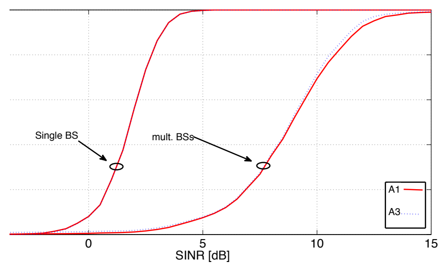

Fig. 2: Cumulative distribution function (CDF) of the SINR

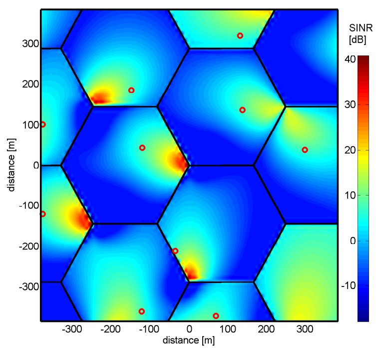

Fig. 3: The SINR as a function of the location. The red circles denote users.

interference decoupled scenario more likely, the interference of the surrounding omnidirectional BSs is increased and the antenna pattern is improved. The interference of adjacent networks can be treated as noise. Hence, a low SNR like in (17) appears more often.

Applying A2, the power constraints are sometimes violated (see Fig. 4). This is always avoided for algorithm A1 even in interference decoupled cases. The main advantage of the iterative solutions A1 and A2 is the lower number of iterations [9] to achieve a balanced SINR compared to the bisection based method A3. The algorithm A3 requires a high accuracy to find an optimal solution. Fig. 5 depicts the optimality ratio of the convex solver based bisection method [5]. The higher the accuracy the more optimal is the solution of the bisection method. An accuracy of $\epsilon = 10^{-9}$ is needed to achieve an optimality of approximately 100%.

VII. Conclusions

This paper presents a solution for the CoMP transmission with cooperation among BSs. The fairness among the jointly scheduled users in a multicell network with per antenna power constraints is desired. The problem is called max-min beamforming problem and has an inner problem which corresponds to a weighted sum power constrained problem (9). This paper further discusses the SINR balancing conditions of the max-min beamforming and gives a discussion of the interference decoupled cases. Based on these considerations, an extended fast direct solution for the max-min beamforming problem with per antenna power constraint is proposed. The algorithm always finds solution without violating the power constraint even if a balanced SINR does not exist and is able to out-perform the well known bisection based method for the same accuracy.

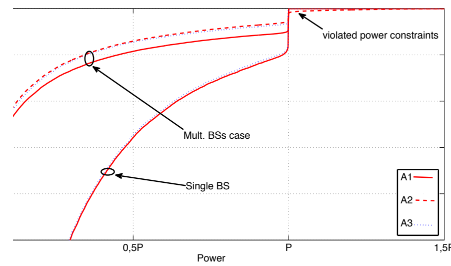

Fig. 4: CDFs of the power per antenna element

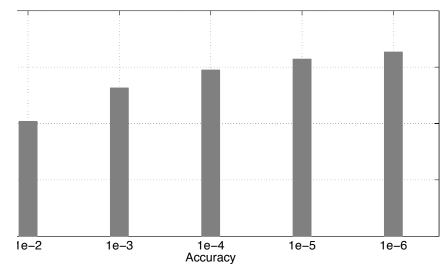

Fig. 5: Optimality of the convex solver based solutions.

VIII. Acknowledgment

This work was supported by UMIC (Ultra High-Speed Mobile Information and Communication) research project at the RWTH-Aachen University.

References

[1] W. Yu and T. Lan, "Transmitter optimization for the multi-antenna downlink with per-antenna power constraints," *IEEE Transactions on Signal Processing*, vol. 55, no. 6, pp. 2646–2660, Jun. 2007.

[2] A. Tölli, H. Pennanen, and P. Komulainen, "SINR Balancing with Coordinated Multi-Cell Transmission," in *IEEE Wireless Communications and Networking Conference, WCNC*, 2009.

[3] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[4] F. Negro, M. Cardone, I. Ghauri, and D. T. M. Slock, "SINR balancing and beamforming for the MISO interference channel," in *22nd IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, Sep. 2011.

[5] G. Dartmann, X. Gong, and G. Ascheid, "Max-min beamforming with cooperative multipoint transmission," in *74th IEEE Vehicular Technology Conference, VTC Fall*, Sep. 2011.

[6] H. W. Je, H. Lee, K. Kwak, S. Choi, Y.-J. Hong, and B. Clerckx, "Long-term channel information-based CoMP beamforming in LTE-advanced systems," in *67th IEEE Vehicular Technology Conference, VTC Spring*, Dec. 2011.

[7] M. Schubert and H. Boche, "Solution of the multiuser downlink beamforming problem with individual SINR constraints," *IEEE Transactions on Vehicular Technology*, vol. 53, no. 1, pp. 18–28, Jan. 2004.

[8] X. Gong, M. Jordan, G. Dartmann, and G. Ascheid, "Max-min beamforming for multicell downlink systems using long-term channel statistics," in *20th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications*, Sep. 2009, pp. 803 –807.

[9] G. Dartmann, W. Afzal, X. Gong, and G. Ascheid, "Low complexity cooperative downlink beamforming in multiuser multicell networks," in *12th IEEE International Conference on Communication Technology, ICCT*, Nov. 2010.

[10] R. Zhang and S. Cui, "Cooperative interference management with MISO beamforming," *IEEE Transactions on Signal Processing*, vol. 58, no. 10, pp. 5450–5458, Oct. 2010.

[11] A. Tölli, M. Codreanu, and M. Juntti, "Linear multiuser MIMO transceiver design with quality of service and per-antenna power constraints," *IEEE Transactions on Signal Processing*, vol. 56, no. 7, pp. 3049–3055, Jul. 2008.