A Rake Finger Grid for asynchronous DS-CDMA Systems using LMMSE Tap Weight Estimation

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***Abstract*—In this paper a Rake finger grid structure for direct-sequence code division multiple access (DS-CDMA) systems is analyzed as an alternative to the conventional Rake structure using individual fingers with timing- and phasor tracking. The grid uses a group of equispaced Rake fingers in the delay domain. Each of the Rake fingers uses linear minimum mean-squared error (LMMSE) tap weight estimation needed for the subsequent maximum-ratio combining (MRC). It is pointed out during the paper that the use of a grid is advantageous compared to the conventional Rake receiver in some scenarios where the channel forms so called clusters, consisting of many unresolvable, closely spaced multipaths. It is also shown that when using fractionally-spaced grid fingers, the task of fine timing tracking can be omitted. Only coarse timing estimation is necessary, which can be performed by a suitable acquisition unit.**

I. Introduction

In conventional Rake receivers used in DS-CDMA systems, a single Rake finger is assigned to each of the identified multipaths after pulse-matched filtering and subsequently, maximum ratio combining (MRC) of all fingers is employed. The main tasks of the receiver's channel tracking unit are on the one hand the estimation of the delays of the multipaths and on the other hand the estimation of the complex valued phasors which are needed for MRC. The delays can be tracked using feedback loops with timing error detectors (TED) and each of the complex valued phasors can be estimated using phasor tracking algorithms; e.g. described in [1] or [2]. After the estimation of these values, they have to be compensated for. Timing is adjusted either by using digital interpolation and decimation operating on at least $T_c/2$-sampling (see [1]) or, without the need for digital interpolation, by choosing the sampling period to be a fraction of $T_c/2$ and then choosing the sample closest to the optimal sampling instant. However, in order to use these simple approaches, the level of inter-chip interference, which also degrades tracking performance, has to be small.

If the multipaths are closely spaced and form a so called cluster, interference between each of them becomes large. Thus, the standard synchronization schemes, as used in the conventional Rake receiver, are not suited to be used in these scenarios with a higher level of inter-chip interference. Within those scenarios, advanced timing tracking schemes [3] must be used to ensure the separation of the different discrete multipaths. The drawback is that the TED used within these schemes are more complex. Also, the inter-chip interference has to be reduced by using interference cancellation schemes e.g. presented in [4]. Here, the estimates of all delay parameters and complex phasors are taken into account to compensate for the interference which leads to a complex cancellation unit especially when the number of multipaths becomes large. Also, the acquisition of the path delay parameters is a very critical task. Appropriate estimators use subspace approaches like the MUSIC algorithm [5], which also bears a higher complexity than standard acquisition schemes. A big problem of this approach is, that the number of discrete multipaths has to be estimated very accurately in advance.

Therefore, the alternative Rake finger grid approach analyzed in this paper is to assign multiple equispaced Rake fingers to that portion of the delay region, which has been identified to contain relevant signal energy. In contrast to the structures presented in [10] or [11], where reduced complexity timing tracking units are still used, the Rake finger grid presented here works on the samples after pulse-matched filtering at the receiver without fine timing tracking but using standard phasor estimation schemes within each finger. The grid utilizes the signal portions of the cluster that are located within its region in the delay domain using subsequent MRC. By doing so, we form an abstraction from the single discrete paths of the channel. The channel may now consist of an arbitrary number of paths, which are closely spaced within a cluster in the delay domain. For a finger spacing equivalent to the symbol duration, the approach has initially been published in [12]. By analyzing the performance sensitivity of the grid structure on fine timing errors, we will show throughout the paper that the task of fine timing estimation (simple *and* advanced) can be omitted in our DS-CDMA system, when using small, fractionally-spaced Rake finger grids.

The paper is structured as follows. Section II presents the system model of our DS-CDMA system. In Section III we introduce the general structure of the Rake grid using independent LMMSE tap weight estimation within each finger. Before conclusions are drawn in Section V, the performance of the grid is analyzed in Section IV.

II. System Model

In our DS-CDMA system BPSK user data d_m and pilot information p_i are transmitted in the real and imaginary part of the effectively transmitted chip

$$a_k = d_{\lfloor \frac{k}{N_d} \rfloor} c_{d,k} \beta_d + j\, p_{\lfloor \frac{k}{N_p} \rfloor} c_{p,k} \beta_p \ , \tag{1}$$

using the spreading factors N_d and N_p for the data- and the pilot symbols, respectively. The respective information is spread and weighted using the spreading sequences c_d, c_p and the factors β_d and β_p. The powers of the transmitted chips a_k, the pilot- and the data symbols as well as the sequences are normalized to one, ($|a_k| = |d_m| = |p_i| = |c_{x,n}| = \beta_d^2 + \beta_p^2 = 1$).

The received signal after root-raised cosine pulse matched filtering can be described as

$$z(nT_s) = \sum_k a_k h(nT_s;\, nT_s - kT_c) + n_c(nT_s) \ , \tag{2}$$

where n_c is the colored noise of power σ_n^2 after root-raised cosine pulse matched filtering and sampling with T_s. The effective channel impulse response (CIR) $h(t\,;\,\tau)$ includes the CIR of the channel $c(t\,;\,\tau)$, the pulse shaping filter $g_T(\tau)$ at the transmitter and the pulse-matched receiver filter $g_R(\tau)$[1].

Now, we can describe the correlator output $z_{x,\,c}(t\,;\,\tau)$ after pulse-matched filtering at the receiver as follows:

$$z_{x,\,c}(m\,;\,n) = \frac{1}{N_x} \sum_{i=mN_x}^{(m+1)N_x-1} b_{x,\,i}^*\, z(nT_s + iT_c)$$

$$= \frac{1}{N_x} \sum_{i=mN_x}^{(m+1)N_x-1} \sum_k b_{x,\,i}^* a_k h(nT_s + iT_c\,;\, nT_s + (i-k)T_c)$$

$$+ \frac{1}{N_x} \sum_{i=mN_x}^{(m+1)N_x-1} b_{x,\,i}^*\, n_c(nT_s + iT_c). \tag{3}$$

Here, N_x is the coherent averaging length of the correlator and "*" denotes the complex conjugate. The variable x can be replaced by 'd' or 'p'. By choosing x we hereby refer to the data or pilot correlator. The correlator sequences $b_{d,\,n}$ and $b_{p,\,n}$ then have to be

$$b_{d,\,n} = \frac{c_{d,n}}{\beta_d} \quad \text{and} \tag{4}$$

$$b_{p,\,n} = j\, \frac{p_{\lfloor \frac{n}{N_p} \rfloor} c_{p,n}}{\beta_p} \ . \tag{5}$$

If we now assume perfect correlation properties of the spreading sequences $c_{x,\,n}$ and when we assume that the channel is slowly varying and therefore not significantly changing during the interval $N_x T_c$, we obtain maximum likelihood (ML) channel estimates $z_{p,\,c}(m\,;\,n) = \hat{h}_{ML}(m\,;\,n)$ (see [1]) at time $t = mN_pT_c$ for the position $\tau = nT_s$ in the delay region at the pilot correlator outputs. Simultaneously, we obtain $z_{d,\,c}(m\,;\,n)$ at the data correlator outputs at time $t = mN_dT_c$ for the position $\tau = nT_s$. When $N_d \neq N_p$ we have to use an appropriate rate matching or interpolation scheme.

III. Rake Grid

The output of a Rake grid using N_f fingers at $t = mN_xT_c$ can now be described by the vector

$$\mathbf{z}_{x,\,c}(m) = \begin{pmatrix} z_{x,\,c}(m\,;\,0) \\ z_{x,\,c}(m\,;\,1) \\ \cdots \\ z_{x,\,c}(m\,;\,N_f-1) \end{pmatrix} . \tag{6}$$

Here, without loss of generality it is assumed that relevant signal energy lies between $\tau = 0$ and $\Delta\tau = (N_f - 1)Ts$. Now, the decision variable

$$y(m) = \mathbf{w}_c^H(m) \cdot \mathbf{z}_{d,\,c}(m) \tag{7}$$

is formed by combining the output of the data correlator bank $\mathbf{z}_{d,\,c}$ using the coefficient vector $\mathbf{w}_c$.

The signal-to-noise ratio (SNR) of y is

$$SNR_y(\mathbf{w}_c) = \frac{\mathbf{w}_c^H \mathbf{R}_h \mathbf{w}_c}{\mathbf{w}_c^H \mathbf{R}_n \mathbf{w}_c} \tag{8}$$

with $\mathbf{R}_h = E\{\mathbf{h}\mathbf{h}^H\}$, the covariance matrix of the true CIR vector $\mathbf{h} = (h(m;0) \cdots h(m;N_f-1))^T$ in the delay region, covered by the Rake grid and $\mathbf{R}_n$, the covariance matrix of the noise at the correlator output. It was shown e.g. in [9] that the optimal combining vector that maximizes the SNR is

$$\mathbf{w}_{c,opt}(m) = \mathbf{R}_n^{-1}\mathbf{h}(m) \ . \tag{9}$$

[1] The impulse response of the channel having L discrete multipaths is generally modelled as

$$c(t;\tau) = \sum_{l=0}^{L-1} \delta(\tau - \tau_l(t)) \cdot c_l(t).$$

leading to the CIR of the effective channel

$$h(t;\tau) = g_T(\tau) * c(t,\tau) * g_R(\tau),$$

where "*" denotes convolution.

Here, in case of white Gaussian noise on the channel, $\mathbf{R}_n$ is known a priori[2]. Applying the principle of synchronized detection [1] we can estimate the CIR vector and use it as if it was the true vector. This then leads to the combiner coefficients

$$\mathbf{w}_c(m) = \mathbf{R}_n^{-1}\hat{\mathbf{h}}(m). \quad (10)$$

In Section II we pointed out that the output of the pilot correlator already represents the ML estimator for the tap weights i.e.

$$\mathbf{z}_{p,\,c}(m) = \hat{\mathbf{h}}_{ML}(m) = \begin{pmatrix} z_{p,\,c}(m\,;\,0) \\ \cdots \\ z_{p,\,c}(m\,;\,N_f-1) \end{pmatrix}. \quad (11)$$

In case of linear minimum mean-squared error (LMMSE) tap weight estimation, the pilot correlator output, i.e. the ML estimates, are additionally to be filtered in the time direction using N_w filter coefficients $\mathbf{w}_n$ for each position n of the CIR. The LMMSE estimator follows directly from the derivation of the achievable rate for a system with pilot transmission on a flat fading channel [7].

In addition to (11), we describe the correlator outputs in direction of time for the CIR tap n, i.e. the delay $\tau = nT_s$ and at time $t = mN_xT_c$ as follows

$$\mathbf{z}^t_{x,\,c}(m\,;\,n) = \begin{pmatrix} z_{x,\,c}(m-(N_w-1)/2\,;\,n) \\ \cdots \\ z_{x,\,c}(m+(N_w-1)/2\,;\,n) \end{pmatrix}, \quad (12)$$

where N_w is the number of correlator output samples taken into account i.e. the size of one filter. This extension of the pilot correlator output in direction of time leads to the N_f x N_w matrix

$$\hat{\mathbf{H}}_{ML}(m) = \begin{pmatrix} \mathbf{z}^{t\,T}_{p,\,c}(m\,;\,0) \\ \cdots \\ \mathbf{z}^{t\,T}_{p,\,c}(m\,;\,N_f-1) \end{pmatrix}. \quad (13)$$

Let now $\hat{\mathbf{h}}_{\mathbf{MLo}}(m) = \mathrm{vec}\{\hat{\mathbf{H}}_{ML}(m)\}$ be the row vector containing the vertically stacked columns of $\hat{\mathbf{H}}_{ML}(m)$. The filtering operations then result in the tap-weight estimates

$$\hat{\mathbf{h}}_w(m) = \mathbf{W}\hat{\mathbf{h}}_{MLo}\ , \quad (14)$$

where

$$\mathbf{W} = \begin{pmatrix} \mathbf{w}_1^H & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{w}_2^H & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{w}_{N_f}^H \end{pmatrix} \quad (15)$$

represents the different filtering operations.

From the literature (e.g. [14], [15]) we know that the LMMSE filter coefficients for for the n^{th} tap are

$$\mathbf{w}_{opt,n} = \mathbf{R}^{-1}_{\mathbf{z}^t_{p,\,c}\mathbf{z}^t_{p,\,c}}\mathbf{r}_{h\,\mathbf{z}^t_{p,\,c}}\ , \quad (16)$$

with $\mathbf{R}_{\mathbf{z}^t_{p,\,c}\mathbf{z}^t_{p,\,c}} = \mathbf{R}_{\mathbf{h}^t\mathbf{h}^t} + \sigma_n^2/N_p\ \mathbf{I}$. The matrix $\mathbf{R}_{\mathbf{h}^t\mathbf{h}^t} = E\{\mathbf{h}^t\mathbf{h}^{tH}\}$ is the covariance matrix of the specific CIR tap $\mathbf{h}^t = (h(m-(N_w-1)/2;n)\cdots h(h(m+(N_w-1)/2;n))^T$ in direction of time; i.e. corresponding to the N_w samples within the filter $\mathbf{w}_n$. The cross-correlation vector $\mathbf{r}_{h\ \mathbf{z}^t_{p,\,c}} = E\{h\ \mathbf{z}^t_{p,\,c}\}$ between the true value of the CIR vector h at the n^{th} tap and the N_w samples within the filter can be written as $\mathbf{r}_{h\ \mathbf{z}^t_{p,\,c}} = E\{h\ \mathbf{h}^t\}$, because the channel and the noise are uncorrelated.

The coefficients depend on the Doppler frequency f_D, or the velocity v of the mobile which is equal for all taps *and* also on the SNR at each tap of the CIR. This implies the use of different sets of filter coefficients for each tap.

In practice, the use of the optimal coefficients w.r.t. SNR is not necessary because the performance sensitivity of an LMMSE Estimator on mismatched SNR is not critical. Instead, filter sets for different SNR classes may be pre-computed and used based on the rough SNR estimation for each tap e.g. obtained from a preceding acquisition unit.

Using the estimate defined in (14) the decision variable becomes

$$y_w(m) = \hat{\mathbf{h}}^H_{MLo}(m)\ \mathbf{W}^H\ \mathbf{R}_n^{-1H}\ \mathbf{z}_{d,\,c}(m). \quad (17)$$

IV. Rake Grid Performance

Determining the bit-error performance of the decision variable after MRC analytically is complicated, even when perfect channel estimation is assumed. Recent results on this topic were presented in [8]. Some approaches taking channel estimation into account were made in [13] but only for uncorrelated taps of equal power.

Therefore, in this section we present Monte-Carlo simulations[3] showing BER results using different tap spacings. The results take effects like e.g. imperfect autocorrelation properties or the imperfect channel estimation described above into account.

The BER results in Figure 1 are plotted versus the parameter "Grid Shift" which denotes the shift of the first Rake finger related to the first path of the channel. The channel used here was the channel "case 3" defined in [16]. It consists of 4 T_c-spaced paths with the delays $\tau_n = [\ 0,1,2,3\ ]\ T_c$ and the relative path powers $p_n = [\ 0,-3,-6,-9\ ]$ dB. The figure shows BER performance curves of Rake grid configurations with

[2] Due to pulse-matched receive filtering, the correlation function of the noise is the raised cosine function.

[3] The results were obtained using the tool CoCentric System Studio from Synopsys Inc.

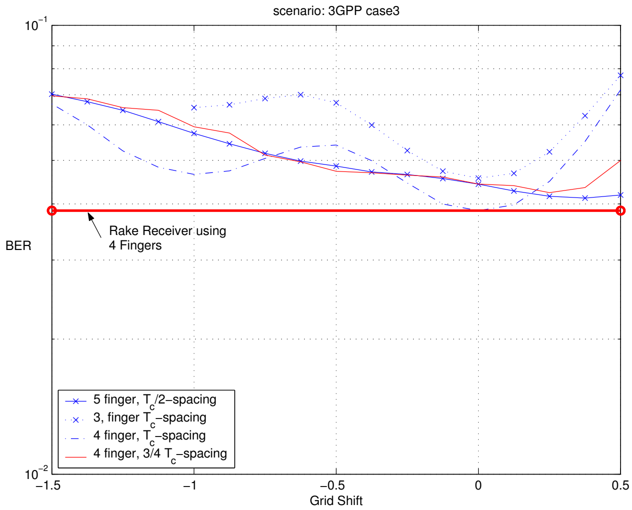

Fig. 1. BER Performance $E_c/N_0 = -11.5$dB, $\beta_p/\beta_d = 0.7333, N_p = 256, N_d = 64, N_w = 19$.

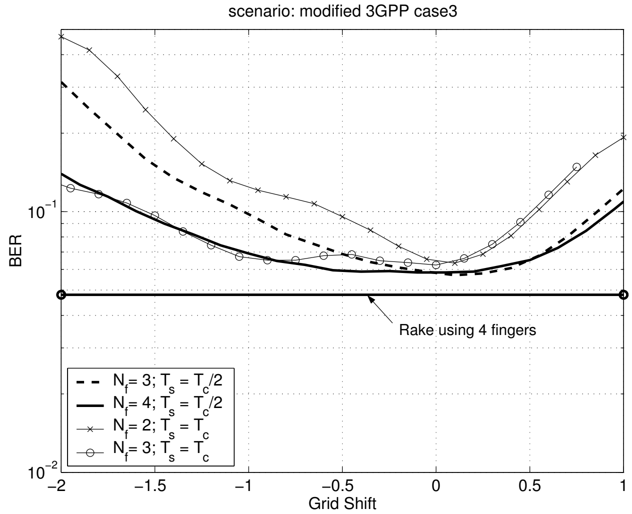

Fig. 2. BER Performance $E_b/N_0 = 4$dB, $\beta_p/\beta_d = 0.7333, N_p = 256, N_d = 64$

different grid-spacings and different numbers of fingers. The curves for a grid spacing of $T_c/2$, 3/4 T_c and the curve for a 3-finger T_c-spaced grid cover a delay region of about $2T_c$. We observe that the performance for the fractionally-spaced structures is less sensitive on the parameter "Grid Shift" than the performance of the T_c-spaced version. The performance of small fractionally-spaced Rake finger groups additionally using a reduced-complexity timing tracking was investigated in [11]. The result above shows that even this timing tracking is not necessary. Instead, only coarse timing synchronization is needed, which can be performed by suitable acquisition schemes. These schemes are less complex than the acquisition schemes needed for the conventional Rake receiver, which uses the advanced tracking methods [6] mentioned above.

In Figure 1, the performance of the conventional Rake receiver using individual fingers is represented by the solid line. The Receiver here uses perfect timing synchronization and LMMSE tap weight estimation ([1], [14], [15]) with perfect interference cancellation. As we can observe, when using the channel "case 3" defined in [16], this performance is also attained by a T_c-spaced grid with a "Grid Shift" of zero. Then, the 4 T_c-spaced paths are exactly sampled by 4 T_c-spaced fingers of the grid, which is identical for the conventional Rake receiver which is operating as described above. Furthermore, the performance of this ideal case is also nearly attained by a fractionally-spaced grid covering a delay region of 2 T_c, whereas we expect a loss for the Rake receiver using the advanced methods, when the synchronization and interference cancellation is no longer perfect.

In the following, we present simulation results using a more smeared channel scenario which is identical to the channel "case 3" defined in [16] but with the relative delays $\tau_n = [0.0, 0.625, 1.125, 1.375]T_c$. Figure 2 again shows BER curves plotted vs. the parameter "Grid Shift". The curves show that for this relatively smeared scenario, the fractionally-spaced grid outperforms the T_c-spaced version in terms of BER, even if the T_c-spaced version is in its best operating point using timing tracking schemes, e.g. presented in [10].

Additionally, the performance of the fractionally-spaced Rake finger grid is less sensitive to timing shifts, especially when a larger number of fingers is used. However, there is a tradeoff when using a large amount of fingers. As long as the collected signal energy overcompensates the collected noise power and the channel estimation error, the performance will increase. When the number of fingers i.e. the covered delay region is larger than the delay spread of the cluster, the performance will obviously decrease, but the structures will be less sensitive to timing shifts. This is exactly what can be observed in Figure 2.

The solid line again represents the performance of a conventional Rake receiver using individual fingers with perfect timing synchronization and LMMSE tap weight estimation with perfect interference cancellation. We observe a performance loss of the grid structures compared to the conventional Rake receiver, but again, we expect a loss for the Rake receiver, when real synchronization and interference cancellation units are used.

V. Conclusion

When small Rake finger grids are used instead of the conventional Rake receiver using advanced methods, summarized in [6], the single finger tracking, i.e. the use of digital interpolation and decimation or alternatively a higher sampling rate of the A/D converter at the receiver input and also the use of the advanced timing synchronization and interference cancellation schemes can be omitted. Each finger only uses a tap weight estimation scheme without the necessity of interference cancellation. Also, relatively simple acquisition algorithms can be used.

The use of a Rake finger grid structure is therefore a tremendous advantage compared to the conventional Rake finger approach using advanced tracking schemes, because implementation complexity is extremely reduced at virtually no performance loss.

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