On the Throughput of Proportional Fair Scheduling with Opportunistic Beamforming for Continuous Fading States

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***Abstract*— We examine the throughput of an opportunistic beamforming system with proportional fair scheduling and show for normally distributed channel fading states that for large numbers of users the average throughput of each user multiplied by the number of users approaches the maximum possible throughput of this user achievable by coherent beamforming, if round robin scheduling was used. Thus, we extend a proof by Viswanath et al. who showed this for discrete fading states. We give the average SNR of the scheduled user (averaged across the fading states) in closed form and the average throughput in form of an integral as a function of the number of transmit antennas and users. Simulations of this system confirm the analytical results. Finally, we show that for a large number of transmit antennas, the probability density function of the SNR of the scheduled user and therefore also the throughput asymptotically approach those of a system with a max SNR scheduler that always transmits to the user having the largest SNR and thus maximizes the total throughput.**

I. Introduction

In a recent publication, P. Viswanath et al. showed that the average throughput in a cellular mobile communication system can be considerably increased if multiuser diversity is exploited [6]. This refers to the fact that there are usually several mobile terminals in a cell waiting for data transmitted over the downlink from a base station (BTS) or central access point. If the mobile terminals estimate their instantaneous channel quality (Signal-to-Noise Ratio, SNR) and feed it back to the BTS, a scheduler in the BTS can use this information to schedule a user that momentarily has an above-average channel quality and can thereby increase the system throughput.

In order to enable all terminals to get their fair share of the channel, it must be ensured that all terminals have a good channel every once in a while. This is either naturally the case, if the channel dynamic is large enough, i.e. the channel state varies fast enough, or a sufficient dynamic must be induced artificially by multiplying the signal with a time varying weight vector. For this principle, Viswanath et al. have coined the term ”Opportunistic Beamforming”.

In [6], the authors prove for discrete channel states that if the number of users goes to infinity and if the distribution of the weight vector is matched to the distribution of the channel states, then the average throughput of each user multiplied by the number of users approaches the maximum possible throughput of this user if round robin scheduling was used.

We consider the case of *continuous* Gaussian channel states. We not only extend the proof by Viswanath et al. to this case, but we derive the pdf and mean of the SNR of the scheduled user and of the average throughput as a function of the number of transmit antennas T and the number of users K. It turns out that we can give closed form expressions of the average scheduled SNR and an integral expression of the average throughput[1], which can be evaluated numerically. Simulations of this system confirm the analytical results. We examine the asymptotic behavior of the proportional fair scheduler both for a large number of antenna elements and a large number of users and show that it approaches the max SNR scheduler in the first case and coherent beamforming with round robin scheduling in the second.

The outline of the paper is as follows. In section II, the system and signal models are introduced. We derive the probability density function (pdf) of the Signal to Noise Ratio (SNR) conditioned on the instantaneous channel state in section III. In section IV we present a modified proportional fair scheduler (PFS) and calculate the pdf and mean of the (unconditioned) SNR with this scheduler. For comparison, we derive the corresponding values for the max SNR scheduler in section V. In sections VI and VII we examine the asymptotic behavior of the PFS scheduler for a large number of antenna elements and users, respectively, followed by a conclusion in section VIII.

II. System Model

We consider a single cell like the one depicted in Fig. 1. One base station equipped with T antennas serves K mobile terminals with one receive antenna each. In each time slot t, a random vector generator produces a $(T \times 1)$ weight vector $\mathbf{w}(t)$. The data sequence $a(t)$ is multiplied by $\mathbf{w}^*(t) = [w_1(t), w_2(t), \ldots, w_T(t)]^H$, where $\mathbf{w}^*$ designates the complex conjugate and $\mathbf{w}^H$ the transposed complex conjugate of $\mathbf{w}$. The resulting vector $a(t)\mathbf{w}^*(t)$ is transmitted. We assume the variance of the transmitted symbols $a(t)$ to be one. Let $\mathbf{h}_k(t) = [h_{1k}(t), h_{2k}(t), \cdots, h_{Tk}(t)]^T$ be the vector of channel coefficients between the antenna array and terminal

[1] In analogy to the publication by Viswanath et al., we use the relation $R = \log_2(1+SNR)$, i.e. we are assuming the use of powerful enough codes such that the data rate achieved in each time slot is given by the Shannon limit.

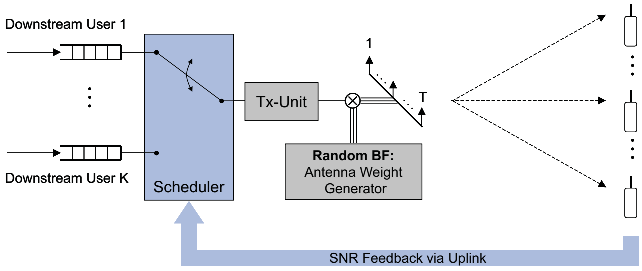

Fig. 1. System Overview

k, i.e. $h_{ik}(t)$ is the channel between terminal k and antenna element i of the BTS. We assume that the channel coefficients are independent $\mathcal{CN}(0,1)$ distributed. (For correlated channels cf. [4].) The signal $r_k(t)$ received by user k can then be written as

$$\begin{aligned} r_k(t) &= h_{1k}(t)w_1^*(t)a(t) + \ldots + h_{Tk}w_T^*(t)a(t) + n(t) \\ &= \mathbf{w}^H(t)\mathbf{h}_k(t)\,a(t) + n(t) \end{aligned} \tag{1}$$

where $n(t)$ is white Gaussian noise with variance σ^2.

In each time slot all users estimate their instantaneous SNR $\gamma_k(t) := \mathbf{w}^H(t)\mathbf{h}_k(t)\mathbf{h}_k^H(t)\mathbf{w}(t)/\sigma^2 = s_k(t)/\sigma^2$, where

$$s_k(t) := \mathbf{w}^H(t)\,\mathbf{H}_k(t)\,\mathbf{w}(t) \tag{2}$$

$$\mathbf{H}_k(t) = \mathbf{h}_k(t)\mathbf{h}_k^H(t) \tag{3}$$

and feed it back to the BTS [3]. A scheduler in the BTS determines which user to transmit to based on all $\gamma_k(t)$. In order to maximize the throughput in the cell it would always schedule the user with the largest instantaneous SNR. In the following, we will call this kind of scheduler a max SNR scheduler.

In this paper, we will focus on what Viswanath et al. call slow fading, where the channel $\mathbf{h}_k(t) = \mathbf{h}_k$ of each user k remains constant for all t (over the latency time scale of interest). We generate the weight vectors $\mathbf{w}$ according to an isotropic distribution [2], which was also proposed by [6] for the independently Gaussian fading channel. An isotropically distributed (i.d.) unit vector $\mathbf{w}$ can be generated by first generating a T-dimensional random vector $\mathbf{z}$ whose elements are independent $\mathcal{CN}(0,1)$ distributed, and then normalizing it according to $\mathbf{w} = \mathbf{z}/\sqrt{\mathbf{z}^H\mathbf{z}}$, i.e.

$$p\left(\mathbf{w}\right) = p\left(\frac{\mathbf{z}}{\sqrt{\mathbf{z}^H\mathbf{z}}}\right). \tag{4}$$

III. Conditional Distribution of the SNR

As we focus on the slow fading case, a max SNR scheduler is not sufficient to guarantee fairness among the users. Users that are at a fading peak are likely to be scheduled all the time, while others that experience deep fades are not scheduled at all.

For this situation, Viswanath proposes to replace the max SNR scheduler by a proportional fair scheduler (PFS) which schedules the users according to their instantaneously supported throughput normalized by their average throughput. We examine a slightly modified version of the PFS algorithm which instead of the throughput considers the instantaneous SNR normalized by the average SNR to make the scheduling decision.

In order to analyze the behavior of this scheduler, we must determine the probability distribution function (pdf) of the SNR γ_k conditioned on the instantaneous (and constant) channel $\mathbf{h}_k$. To achieve this, we must analyze the distribution of s_k conditioned on $\mathbf{H}_k$. As $\mathbf{H}_k$ is obviously hermitian, we can decompose it into

$$\mathbf{H}_k = \mathbf{U}\,\mathbf{D}_k\,\mathbf{U}^H \tag{5}$$

where

$$\mathbf{D}_k = \begin{pmatrix} H_k & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \tag{6}$$

$$H_k = \mathbf{h}_k^H\mathbf{h}_k \tag{7}$$

and $\mathbf{U}$ is a unitary matrix. Thus

$$\begin{aligned} s_k &= \mathbf{w}^H\,\mathbf{U}\,\mathbf{D}_k\,\mathbf{U}^H\,\mathbf{w} && (8) \\ &= \tilde{\mathbf{w}}^H\,\mathbf{D}_k\,\tilde{\mathbf{w}} && (9) \\ &= H_k\,|\tilde{w}_1|^2 && (10) \end{aligned}$$

where $\tilde{\mathbf{w}} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_T)^T = \mathbf{U}^H\,\mathbf{w}$ is also an i.d. unit vector. (This follows from the definition of an i.d. unit vector, cf. [2].)

We finally get the following expression for the SNR:

$$\gamma_k = \frac{H_k\,|\tilde{w}_1|^2}{\sigma^2} \tag{11}$$

As $\tilde{\mathbf{w}}$ is an i.d. unit vector, it must be distributed according to (4). Thus we can express the pdf of $|\tilde{w}_1|^2$ by

$$p(|\tilde{w}_1|^2) = (T-1)(1-|\tilde{w}_1|^2)^{T-2}; \qquad 0 \le |\tilde{w}_1|^2 \le 1 \tag{12}$$

A transformation of variables yields the pdf of the SNR conditioned on the channel:

$$p(\gamma_k|H_k) = (T-1)\left(1-\frac{\gamma_k\sigma^2}{H_k}\right)^{T-2}\frac{\sigma^2}{H_k}; \qquad 0 \le \gamma_k \le \frac{H_k}{\sigma^2} \tag{13}$$

and the mean SNR of user k is given by

$$E(\gamma_k|H_k) = \frac{H_k}{T\sigma^2}. \tag{14}$$

IV. A Fair Scheduler

In the modified version of the PFS algorithm that we examine here, the instantaneous SNR is normalized by its mean. Thus, the decision variable for the scheduler is

$$\begin{aligned} d_k &:= \gamma_k/E(\gamma_k|H_k) && (15) \\ &= \frac{T\sigma^2\gamma_k}{H_k} && (16) \end{aligned}$$

The pdf of this decision variable is

$$p(d_k|H_k) = \frac{T-1}{T}\left(1-\frac{d_k}{T}\right)^{T-2}; \qquad 0 \le d_k \le T \tag{17}$$

which is no function of the instantaneous channel $\mathbf{h}_k$ and is therefore identical for all users. If they were also independent, we could obtain the pdf of the maximum of K decision variables

$$p(d_{max}) = \frac{K(T-1)}{T}\left(1-\left(1-\frac{d}{T}\right)^{T-1}\right)^{K-1}\left(1-\frac{d}{T}\right)^{T-2} \quad (18)$$

Undoing the normalization, we obtain the pdf of the scheduled SNR, conditioned on the event that user k is scheduled:

$$p(\gamma_{PFS}|H_k,k) = \frac{K(T-1)\sigma^2}{H_k}\left(1-\left(1-\frac{\gamma\sigma^2}{H_k}\right)^{T-1}\right)^{K-1} \cdot\left(1-\frac{\gamma\sigma^2}{H_k}\right)^{T-2} \quad (19)$$

Unfortunately, if the channel is constant, the SNR and therefore also the decision variables are *not* independent. Thus, (18) and (19) do not give the actual pdfs. However, we are not interested in the behavior of the system for a *specific* realization of the channel, but rather in the mean behavior. Thus, we must average over the distribution of the channel vectors $\mathbf{h}_k$. But as we assume independently fading Gaussian channel vectors, the averaging corrects the simplifications introduced in (18) and (19).

After averaging over the distribution of H_k, we get

$$p(\gamma_{PFS}) = K\sigma^2 e^{-\gamma\sigma^2}\int_0^\infty \frac{1}{(T-2)!} \cdot\left(1-\left(\frac{x}{x+\gamma\sigma^2}\right)^{T-1}\right)^{K-1} x^{T-2}\, e^{-x}\, dx \quad (20)$$

$$= \frac{K(\sigma^2)^T\gamma^{T-1}}{(T-2)!}\sum_{i=0}^{K-1}\binom{K-1}{i} \cdot\left[\sum_{j=0}^{i(T-1)}\binom{(i+1)T-i-2}{j+T-2}(-1)^{i+j+T}E_j(\sigma^2\gamma) + \sum_{j=1}^{T-2}\binom{(i+1)T-i-2}{T-2-j}(-1)^{i+T-j}\alpha_j(\sigma^2\gamma)\right] \quad (21)$$

where E_j and α_j are exponential integrals [1] defined as

$$E_j(z) = \int_1^\infty \frac{e^{-zt}}{t^j}\, dt \quad (22)$$

$$\alpha_j(z) = \int_1^\infty e^{-zt}t^j\, dt \quad (23)$$

Its expectation is

$$E(\gamma_{PFS}) = \frac{T}{\sigma^2}\left(1-\frac{1}{T-1}\frac{\Gamma(K+1)\Gamma(\frac{1}{T-1})}{\Gamma(K+1+\frac{1}{T-1})}\right) \quad (24)$$

where Γ is the Gamma function [1].

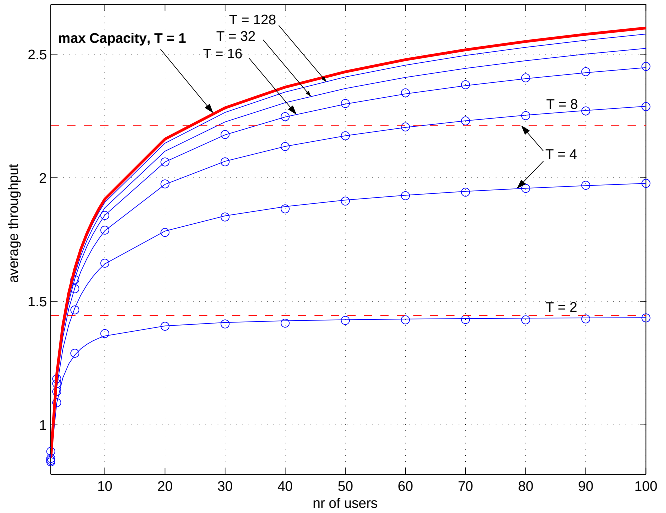

Fig. 2. Average throughput in b/s/Hz averaged over slow Rayleigh fading at 0 dB SNR with the proportional fair scheduling algorithm. The circles show the simulated values, the dashed lines denote the beamforming throughput R^{bf}.

By numeric integration we finally obtain the average throughput of the modified proportional fair scheduler for any number of antennas and users, which is plotted in Fig. 2 along with simulated values for a mean SNR of all users of 0 dB ($\sigma^2 = 1$).

V. Max SNR Scheduler

The average throughput in the cell can be maximized if the PFS scheduler is replaced by a max SNR scheduler which transmits always to the user having the largest SNR. For this system, we can calculate the pdf of the scheduled SNR by the following steps:

First we notice that the distribution of the effective channel $\mathbf{w}^H\,\mathbf{h}_k$ is $\mathcal{CN}(0,1)$. Therefore the pdf of the SNR is

$$p(\gamma_k) = \sigma^2 e^{-\sigma^2\gamma_k}, \qquad \gamma_k \geq 0 \quad (25)$$

for each user k. The cumulative distribution function (cdf) is thus

$$F(\gamma_k) = 1 - e^{-\sigma^2\gamma_k}, \qquad \gamma_k \geq 0 \quad (26)$$

The cdf of the scheduled SNR is therefore

$$F(\gamma_{max}) = \left(1-e^{-\sigma^2\gamma_{max}}\right)^K, \qquad \gamma_{max} \geq 0 \quad (27)$$

and the pdf

$$p(\gamma_{max}) = \sigma^2 K e^{-\sigma^2\gamma_{max}}\left(1-e^{-\sigma^2\gamma_{max}}\right)^{K-1}, \qquad \gamma_{max} \geq 0 \quad (28)$$

with mean

$$E(\gamma_{max}) = \frac{1}{\sigma^2}\sum_{n=1}^{K}\frac{1}{n} \quad (29)$$

Again we can obtain the average throughput by numeric integration. This curve is also plotted in Fig. 2 (as ”max Capacity”).

VI. Asymptotic behavior of the PFS scheduler for large T

If we take a closer look at Fig. 2, we notice that the average throughput of the PFS scheduler approaches that of the max SNR scheduler as the number of antenna elements T goes to infinity. We can even proof the following, stronger statement

$$\lim_{T\to\infty} p(\gamma_{pfs}) = \sigma^2 K e^{-\sigma^2\gamma}\left(1-e^{-\sigma^2\gamma}\right)^{K-1} = p(\gamma_{max}) \tag{30}$$

which implies that all moments including mean and variance of the PFS scheduler approach those of the max SNR scheduler for large T.

When we compare (20) and (28), we see that (30) is obviously true if and only if

$$\begin{aligned}
&\lim_{T\to\infty}\int_0^\infty \frac{1}{T!}\left(1-\left(\frac{x}{x+y}\right)^{T+1}\right)^K x^T\, e^{-x}\, dx \\
&= \lim_{T\to\infty}\sum_{i=0}^{K}\binom{K}{i}\int_0^\infty \frac{x^T e^{-x}}{T!}\,(-1)^i\left(\frac{x}{x+y}\right)^{i(T+1)} dx \\
&= \sum_{i=0}^{K}\binom{K}{i}(-1)^i\, e^{-iy} = \left(1-e^{-y}\right)^K
\end{aligned} \tag{31}$$

Thus it remains to show that

$$\lim_{T\to\infty}\frac{1}{T!}\int_0^\infty\left(\frac{x}{x+y}\right)^{i(T+1)} x^T\, e^{-x}\, dx = e^{-iy} \tag{32}$$

As the proof is rather lengthy, we will only give a sketch: First we can show that

$$\begin{aligned}
e^{-1}\sum_{i=0}^{T}\frac{1}{i!} &\le \frac{1}{T!}\int_1^\infty\left(\frac{(z+iy)^{i+1}}{(z+(i+1)\,y)^i}\right)^T e^{-z}\, dz \\
&\le 1+\frac{1}{\lceil T/2\rceil}e^c+\frac{1}{T!}(1+c)^T\, e^{-1}
\end{aligned} \tag{33}$$

with $c \geq 0$. Because

$$\lim_{T\to\infty} e^{-1}\sum_{i=0}^{T}\frac{1}{i!} = e^{-1}e = 1 \tag{34}$$

and

$$\lim_{T\to\infty} 1+\frac{1}{\lceil T/2\rceil}e^c+\frac{1}{T!}(1+c)^T\, e^{-1} = 1 \tag{35}$$

we obtain

$$\lim_{T\to\infty}\frac{1}{T!}\int_1^\infty\left(\frac{(z+iy)^{i+1}}{(z+(i+1)\,y)^i}\right)^T e^{-z}\, dz = 1 \tag{36}$$

On the other hand we have

$$\begin{aligned}
0 &\le \frac{1}{T!}\int_{-iy}^{1}\left(\frac{(z+iy)^{i+1}}{(z+(i+1)\,y)^i}\right)^T e^{-z}\, dz \\
&= \frac{1}{T!}\int_0^{1+iy}\left(\frac{x^{i+1}}{(x+y)^i}\right)^T e^{-x}\, e^{iy}\, dx \\
&\le e^{iy}\frac{1}{T!}\int_0^{1+iy} x^T\, dx \\
&= e^{iy}\frac{1}{(T+1)!}\,(1+iy)^{T+1}
\end{aligned} \tag{37}$$

As

$$\lim_{T\to\infty} e^{iy}\frac{1}{(T+1)!}\,(1+iy)^{T+1} = 0, \tag{38}$$

we see that

$$\lim_{T\to\infty}\frac{1}{T!}\int_{-iy}^{1}\left(\frac{(z+iy)^{i+1}}{(z+(i+1)\,y)^i}\right)^T e^{-z}\, dz = 0, \tag{39}$$

and finally

$$\lim_{T\to\infty}\frac{1}{T!}\int_{-iy}^{\infty}\left(\frac{(z+iy)^{i+1}}{(z+(i+1)\,y)^i}\right)^T e^{-z}\, dz = 1 \tag{40}$$

which is equivalent to

$$\lim_{T\to\infty}\frac{1}{T!}\int_0^\infty\left(\frac{x^{i+1}}{(x+y)^i}\right)^T e^{-x}\, dx = e^{-iy} \tag{41}$$

Now we show that

$$\begin{aligned}
&\frac{1}{T!}\int_0^\infty\left(\frac{x}{x+y}\right)^{i(T+1)} x^T\, e^{-x}\, dx \\
&= \frac{1}{T!}\int_0^\infty\left(\frac{x^{i+1}}{(x+y)^i}\right)^T e^{-x}\, dx \\
&+ \sum_{j=1}^{i}\binom{i}{j}(-1)^j\, y^j\,\frac{1}{T!}\int_0^\infty\frac{x^{(i+1)T}}{(x+y)^{iT+j}}\, e^{-x}\, dx
\end{aligned} \tag{42}$$

We notice that

$$\begin{aligned}
0 &\le \frac{1}{T!}\int_0^\infty\frac{x^{(i+1)T}}{(x+y)^{iT+j}}\, e^{-x}\, dx \le \frac{1}{T!}\int_0^\infty x^{T-j}\, e^{-x}\, dx \\
&= \frac{(T-j)!}{T!}
\end{aligned} \tag{43}$$

Because

$$\lim_{T\to\infty}\frac{(T-j)!}{T!} = 0 \quad \forall\, 1\le j\le T \tag{44}$$

we obtain

$$\lim_{T\to\infty}\frac{1}{T!}\int_0^\infty\frac{x^{(i+1)T}}{(x+y)^{iT+j}}\, e^{-x}\, dx = 0 \quad \forall\, 1\le j\le T \tag{45}$$

and thus

$$\begin{aligned}
&\lim_{T\to\infty}\frac{1}{T!}\int_0^\infty\left(\frac{x}{x+y}\right)^{i(T+1)} x^T\, e^{-x}\, dx \\
&= \lim_{T\to\infty}\frac{1}{T!}\int_0^\infty\left(\frac{x^{i+1}}{(x+y)^i}\right)^T e^{-x}\, dx \\
&= e^{-iy}
\end{aligned} \tag{46}$$

which completes the proof.

VII. Asymptotic behavior of the PFS scheduler for large K

In this section we analyze the properties of the PFS scheduler as the number of users approaches infinity. As the term in parentheses in (24) approaches 1 for $K \to \infty$ we obtain

$$\lim_{K\to\infty} E(\gamma_{PFS}) = T/\sigma^2 \tag{47}$$

which corresponds to the mean SNR of a user, if the basestation knew the exact channel and formed a beam towards it. This means that for a large number of users the mean scheduled SNR approaches the maximum possible SNR of the beamforming constellation.

In order to examine the behavior of the average throughput as K approaches infinity, we use (19) to express the mean throughput, see (50) at the bottom of the page.

Thus

$$\begin{aligned}
&-\log_2(e)\int_0^1 \left(1-(1-z)^{T-1}\right)^K dz \\
&\quad\cdot\int_0^\infty \frac{H_k}{\sigma^2(T-1)!}e^{-H}H^{T-1}\,dH \\
&= -\frac{T}{\sigma^2}\log_2(e)\int_0^1 \left(1-(1-z)^{T-1}\right)^K dz && (48)\\
&\leq E(R_{PFS})-\int_0^\infty \frac{\log_2(1+H_k/\sigma^2)}{(T-1)!}e^{-H}H^{T-1}\,dH \\
&\leq 0 && (49)
\end{aligned}$$

As the integral in (48) vanishes for $K \to \infty$, we have

$$\begin{aligned}
\lim_{K\to\infty} KR^{(K)} &= \lim_{K\to\infty} E(R_{PFS}) \\
&= \int_0^\infty \frac{\log_2(1+H/\sigma^2)}{(T-1)!}e^{-H}H^{T-1}\,dH \\
&= R^{bf}
\end{aligned}$$

where $R^{(K)}$ is the average throughput a user achieves in the opportunistic beamforming system when there are K users in the cell, and R^{bf} is the throughput that a user achieves when it is in the coherent beamforming constellation, i.e., when $\mathbf{w} = \mathbf{h}_k/\sqrt{\mathbf{h}_k^H\mathbf{h}_k}$ and thus its instantaneous SNR is H_k/σ^2.

The first equation follows from the fact that the probability that a user is scheduled is 1/K (all users have identically distributed d_k), which completes the proof that each user's throughput approaches the throughput achieved by coherent beamforming if K goes to infinity.

The behavior of the max SNR scheduler in this case has been examined by Sharif and Hassibi [5]. They find

$$\lim_{K\to\infty}\frac{E(R_{max})}{\log_2\log K} = 1 \tag{51}$$

VIII. Conclusion

In order to increase the spectral efficiency of cellular wireless systems, all available forms of diversity should be exploited. One novel concept to exploit multiuser diversity is random beamforming and smart scheduling as proposed in [6], where the signal is multiplied by a time varying weight vector to increase the natural dynamic of the channel and a scheduler in the basestation determines the user to transmit to based on the instantaneous SNR fed back from the users.

In this paper, we analyzed two schedulers, the max SNR scheduler which maximizes the total average throughput by always transmitting to the user having the best channel, and a modified version of the proportional fair scheduler which guarantees that all users are scheduled the same number of times. We derived the pdf of the scheduled SNR, found closed form expressions for its mean, calculated the average throughput by numerical integration and confirmed the analytical results by means of simulation. Finally, we examined the asymptotic behavior of the proportional fair scheduler and showed that it approaches the max SNR scheduler if the number of antenna elements becomes large and coherent beamforming with round robin scheduling if there are many users in the cell.

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$$\begin{aligned}
&E(R_{PFS}) \\
&= \int_0^\infty\int_0^{H_k/\sigma^2}\log_2(1+\gamma)\frac{K(T-1)\sigma^2}{H_k}\left(1-\left(1-\frac{\gamma\sigma^2}{H_k}\right)^{T-1}\right)^{K-1}\left(1-\frac{\gamma\sigma^2}{H_k}\right)^{T-2}d\gamma\,\frac{e^{-H}H^{T-1}}{(T-1)!}\,dH \\
&= \int_0^\infty\left[\log_2(1+H_k/\sigma^2)-\log_2(e)\int_0^1\frac{H_k}{\sigma^2+H_kz}\left(1-(1-z)^{T-1}\right)^K dz\right]\frac{1}{(T-1)!}e^{-H}H^{T-1}\,dH \qquad (50)
\end{aligned}$$