Proportional Fair Multibeam Scheduling with Opportunistic Beamforming for Broadcast Channels

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Abstract

We propose an opportunistic beamforming system in which parallel data streams are transmitted to several users at the same time and derive the probability density function of the SINR if a max SINR scheduler is used, which maximizes the total average throughput by always transmitting to the user having the best channel. In slowly fading scenarios this scheduler is not sufficient to support fairness among the users. Therefore we propose a novel multibeam scheduler which guarantees fairness in terms of equal transmission time for all users. For this scheduler, we give the average SINR of the scheduled user (averaged across the fading states) in closed form and the average throughput in form of an integral as a function of the number of transmit antennas and users. Simulations of this system confirm the analytical results. Finally, we show that the average throughput in the multibeam system approaches that of coherent beamforming with round robin scheduling if the number of users is large.

I. Introduction

In a recent publication, P. Viswanath et al. showed that the average throughput in a cellular mobile communication system can be considerably increased if multiuser diversity is exploited [8]. This refers to the fact that there are usually several mobile terminals in a cell waiting for data transmitted over the downlink from a base station (BTS) or central access point. If the mobile terminals estimate their instantaneous channel quality (Signal-to-Noise Ratio, SNR) and feed it back to the BTS, a scheduler in the BTS can use this information to schedule a user that momentarily has an above-average channel quality and can thereby increase the system throughput.

In order to enable all terminals to get their fair share of the channel, it must be ensured that all terminals have a good channel every once in a while. This is either naturally the case, if the channel dynamic is large enough, i.e. the channel state varies fast enough, or a sufficient dynamic must be induced artificially by multiplying the signal with a time varying weight vector. For this principle, Viswanath et al. have coined the term ”Opportunistic Beamforming”.

In [5], we examined two schedulers for an opportunistic single beam system where in each time slot one data packet is transmitted to one single user. In this paper we examine the extension of our approach in [5] to a broadcasting or multibeam system. This means that instead of transmitting only one data stream to one single user, we now serve multiple users at the same time (still assuming that each receiver is equipped with only one antenna). We develop a novel proportional fair multibeam scheduler and derive the pdf of the SINR of the scheduled user and the average throughput as a function of the number of transmit antennas

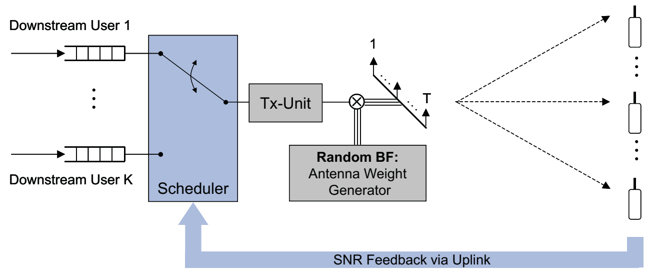

Fig. 1. System Overview

T and the number of users K. It turns out that we can give closed form expressions of the average scheduled SINR and an integral expression of the average throughput[1], which can be evaluated numerically. Simulations of this system confirm the analytical results. We examine the asymptotic behavior of the proportional fair multibeam scheduler for a large number of users and show that it approaches coherent beamforming with round robin scheduling.

The outline of the paper is as follows. In section II, the system and signal models are introduced. We derive the probability density function (pdf) of the scheduled SINR for the max SINR scheduler in section III. In order to design a novel proportional fair multibeam scheduler in section V, we derive the pdf of the SINR conditioned on the instantaneous channel state in section IV. In section VI we show some numerical results and examine the asymptotic behavior of the PFS scheduler for a large number of users in section VII, followed by a conclusion in section VIII.

II. System Model

We consider a single cell like the one depicted in Fig. 1. One base station equipped with T antennas serves K mobile terminals with one receive antenna each. In this paper we consider only the case where the number of beams is equal to the number of transmit antennas T. The data sequence can thus be written as a sequence of T-dimensional column vectors $\mathbf{a}(t)$ with $E\{\mathbf{a}^H(t)\,\mathbf{a}(t)\} = 1$, so the transmitted power does not increase with the number of antennas. This sequence is multiplied by $(T \times T)$ weight matrices $\mathbf{W}^*(t) = [\mathbf{w}_1^*(t), \mathbf{w}_2^*(t), \cdots, \mathbf{w}_T^*(t)]$. The resulting vector $\mathbf{W}^*(t)\,\mathbf{a}(t)$ is transmitted. The signal $r_k(t)$ received by user k can then be written as

$$
\begin{aligned}
r_k(t) &= \mathbf{h}_k^T(t)\,\mathbf{W}^*(t)\,\mathbf{a}(t) + n(t) \\
&= \mathbf{h}_k^T(t)\,[\mathbf{w}_1^*(t), \mathbf{w}_2^*(t), \cdots, \mathbf{w}_T^*(t)] \begin{pmatrix} a_1(t) \\ a_2(t) \\ \vdots \\ a_T(t) \end{pmatrix} + n(t) \\
&= \mathbf{h}_k^T(t)\ [\mathbf{w}_1^*(t)\,a_1(t) + \mathbf{w}_2^*(t)\,a_2(t) + \ldots + \mathbf{w}_T^*(t)\,a_T(t)] + n(t) \\
&= a_1(t)\,\mathbf{w}_1^H(t)\,\mathbf{h}_k(t) + a_2(t)\,\mathbf{w}_2^H(t)\,\mathbf{h}_k(t) + \ldots + a_T(t)\,\mathbf{w}_T^H(t)\,\mathbf{h}_k(t) + n(t) \quad (1)
\end{aligned}
$$

Thus the received signal power of data stream i is $\mathbf{w}_i^H(t)\,\mathbf{h}_k(t)\,\mathbf{h}_k^H(t)\,\mathbf{w}_i(t)/T$, which is identical to $[\mathbf{S}_k(t)]_{ii}\,/T$, where

$$\mathbf{S}_k(t) := \mathbf{W}^H(t)\,\mathbf{H}_k(t)\,\mathbf{W}(t) \quad (2)$$

$$\mathbf{H}_k(t) = \mathbf{h}_k(t)\,\mathbf{h}_k^H(t) \quad (3)$$

[1]In analogy to the publication by Viswanath et al., we use the relation $R = \log_2(1 + SINR)$, i.e. we are assuming the use of powerful enough codes such that the data rate achieved in each time slot is given by the Shannon limit.

and the subscript ii denotes the element in column i and row i. The SINR of data stream i for user k therefore amounts to

$$\gamma_{k,i} = \frac{\frac{1}{T}\left[\mathbf{S}_k(t)\right]_{ii}}{\frac{1}{T}\sum_{\substack{j=1\\j\neq i}}^{T}\left[\mathbf{S}_k(t)\right]_{jj} + \sigma^2} \tag{4}$$

In order to calculate the pdf of the SINR and thus the average throughput, we must therefore analyze the distribution of the elements on the main diagonal of $\mathbf{S}_k(t)$.

In each time slot all users estimate the instantaneous SINRs of all data streams and feed them back to the BTS. A scheduler in the BTS then determines for each data stream i, which user to transmit to based on all $\gamma_{k,i}(t)$. As the following considerations apply to all data streams i, we will drop the index in the remainder of this paper, except where it is needed to avoid confusion.

In this paper, we will focus on what Viswanath et al. call slow fading, where the channel $\mathbf{h}_k(t) = \mathbf{h}_k$ of each user k remains constant for all t (over the latency time scale of interest). As in [7], we assume that the weight matrices $\mathbf{W}$ are generated according to an isotropic distribution [2], which is the natural extension of the single beam case [5], [8]. An isotropically distributed (i.d.) unitary matrix can be generated by first generating a $T \times T$ random matrix $\mathbf{Y}$ whose elements are independent circularly symmetric complex normal $\mathcal{CN}(0,1)$, and then perform the QR decomposition $\mathbf{Y} = \mathbf{W}\,\mathbf{R}$, where $\mathbf{R}$ is upper triangular and $\mathbf{W}$ is an i.d. unitary matrix. (See [2] for a proof.)

If $\mathbf{W}$ is i.d., then the column vectors $\mathbf{w}_1(t), \mathbf{w}_2(t), \cdots, \mathbf{w}_T(t)$ are orthonormal and identically distributed with

$$p\left(\mathbf{w}\right) = p\left(\frac{\mathbf{n}}{\sqrt{\mathbf{n}^H\mathbf{n}}}\right). \tag{5}$$

where $\mathbf{n}$ is a T-dimensional column vector whose elements are independent $\mathcal{CN}(0,1)$.

III. Max SNR Scheduler

The average throughput in the cell can be maximized if a max SINR scheduler which transmits always to the users having the largest SINR is used. For this system, we can calculate the pdf of the scheduled SINR by the following steps.

First we notice that the distribution of the effective channel $\mathbf{h}_k^T\mathbf{W}^*$ is $\mathcal{CN}(0, I_T)$. Therefore the power of the beams is independently chi-square distributed with two degrees of freedom (real and imaginary parts) and the pdf of the SINR is

$$p(\gamma_k) = \frac{e^{-T\sigma^2\gamma_k}}{(1+\gamma_k)^T}\left(T\sigma^2(1+\gamma_k) + T - 1\right), \qquad \gamma_k \geq 0 \tag{6}$$

for each user k (cf. [7]). The cumulative distribution function (cdf) is thus

$$F(\gamma_k) = 1 - \frac{e^{-T\sigma^2\gamma_k}}{(1+\gamma_k)^{T-1}}, \qquad \gamma_k \geq 0 \tag{7}$$

The cdf of the scheduled SNR is therefore

$$F(\gamma_{max}) = \left(1 - \frac{e^{-T\sigma^2\gamma_{max}}}{(1+\gamma_{max})^{T-1}}\right)^K, \qquad \gamma_{max} \geq 0 \tag{8}$$

and the pdf

$$p(\gamma_{max}) = K\left(1 - \frac{e^{-T\sigma^2\gamma_{max}}}{(1+\gamma_{max})^{T-1}}\right)^{K-1}\frac{e^{-T\sigma^2\gamma_{max}}}{(1+\gamma_{max})^T}\left(T\sigma^2(1+\gamma_{max}) + T - 1\right) \tag{9}$$

IV. Conditional Distribution of the SNR

As we focus on the slow fading case, a max SINR scheduler is not sufficient to guarantee fairness among the users. Users that are at a fading peak are likely to be scheduled all the time, while others that experience deep fades are not scheduled at all.

In order to develop a scheduler that guarantees fairness among the users in terms of equal transmission time, we must determine the pdf of the SINR $\gamma_{k,i}$ conditioned on the instantaneous (and constant) channel $\mathbf{h}_k$. To achieve this, we must analyze the distribution of the elements on the main diagonal of $\mathbf{S}_k$ conditioned on $\mathbf{H}_k$ (cf. (4)). As $\mathbf{H}_k$ is obviously hermitian, we can decompose it into

$$\mathbf{H}_k = \mathbf{U}_k \, \mathbf{D}_k \, \mathbf{U}_k^H \tag{10}$$

where

$$\mathbf{D}_k = \begin{pmatrix} H_k & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix} \tag{11}$$

$$H_k = \mathbf{h}_k^H \mathbf{h}_k \tag{12}$$

and $\mathbf{U}_k$ is a unitary matrix.

Thus

$$\mathbf{S}_k = \mathbf{W}^H \, \mathbf{U}_k \, \mathbf{D}_k \, \mathbf{U}_k^H \, \mathbf{W} \tag{13}$$

$$= \tilde{\mathbf{W}}_k^H \, \mathbf{D}_k \, \tilde{\mathbf{W}}_k \tag{14}$$

where $\tilde{\mathbf{W}}_k = \left(\tilde{\mathbf{w}}_1^{(k)}, \tilde{\mathbf{w}}_2^{(k)}, \ldots, \tilde{\mathbf{w}}_T^{(k)} \right) = \mathbf{U}_k^H \, \mathbf{W}$ is also i.d. (This follows from the definition of an i.d. matrix, cf. [2].)

Therefore the main diagonal elements of $\mathbf{S}_k$ can be written as

$$[\mathbf{S}_k]_{ii} = H_k \, \left| \tilde{w}_{i1}^{(k)} \right|^2 \tag{15}$$

where $\tilde{w}_{i1}^{(k)}$ denotes the first element of the column vector $\tilde{\mathbf{w}}_i^{(k)}$.

We finally get the following expression for the SINR:

$$\gamma_{k,i} = \frac{\frac{1}{T} H_k \, \left| \tilde{w}_{i1}^{(k)} \right|^2}{\frac{1}{T} \sum_{\substack{j=1 \\ j \neq i}}^{T} H_k \, \left| \tilde{w}_{j1}^{(k)} \right|^2 + \sigma^2} \tag{16}$$

$$= \frac{H_k \, \left| \tilde{w}_{i1}^{(k)} \right|^2}{H_k - H_k \, \left| \tilde{w}_{i1}^{(k)} \right|^2 + T\sigma^2} \tag{17}$$

where the last equation results from

$$\sum_{j=1}^{T} \left| \tilde{w}_{j1}^{(k)} \right|^2 = 1 \tag{18}$$

because $\tilde{\mathbf{W}}_k$ is unitary.

As $\tilde{\mathbf{W}}_k$ is i.d., its column vectors are distributed according to (5). Thus we can calculate the pdf of $\left| \tilde{w}_{i1}^{(k)} \right|^2$ (cf. [8], p. 1282):

$$p(\left| \tilde{w}_{i1}^{(k)} \right|^2) = (T-1)(1 - \left| \tilde{w}_{i1}^{(k)} \right|^2)^{T-2}; \qquad 0 \leq \left| \tilde{w}_{i1}^{(k)} \right|^2 \leq 1 \tag{19}$$

A transformation of variables yields the pdf of the SINR conditioned on the channel:

$$p(\gamma_{k,i}|H_k) = (T-1)\left(\frac{H_k - T\sigma^2\gamma_{k,i}}{H_k(1+\gamma_{k,i})}\right)^{T-2}\frac{H_k + T\sigma^2}{H_k(1+\gamma_{k,i})^2}; \qquad 0 \le \gamma_{k,i} \le \frac{H_k}{T\sigma^2} \tag{20}$$

and the mean SINR of user k is given by

$$\begin{aligned} E(\gamma_{k,i}|H_k) &= (T-1)\left[\sum_{i=0}^{T-3}(-1)^i\frac{(T\sigma^2)^i}{(T-1-i)(T-2-i)H_k^i} + (-1)^{T-1}\frac{(T\sigma^2)^{T-2}}{H_k^{T-2}}\right. \\ &\quad \left. +(-1)^{T-2}\frac{(T\sigma^2)^{T-2}(H_k+T\sigma^2)}{H_k^{T-1}}\ln\left(1+\frac{H_k}{T\sigma^2}\right)\right] \end{aligned} \tag{21}$$

V. Proportional Fair Multibeam Scheduling

In the literature many definitions of fairness can be found. Our aim is to schedule each user the same number of times. This is obviously guaranteed if the pdf of the variable that the scheduling decision is based on is identical for all users (conditioned on the channel). One possible solution is to choose the decision variable to be uniformly distributed between 0 and 1. This can be achieved if we choose $d_k = F(\gamma_k|H_k)$ (cf. [3]) where F is the cumulative distribution function.

From (20) we obtain

$$\begin{aligned} d_k &:= F(\gamma_k|H_k) && (22)\\ &= 1-\left(1-\frac{H_k+T\sigma^2}{H_k(1+\gamma_k)}\gamma_k\right)^{T-1}; \qquad 0\le\gamma_k\le\frac{H_k}{T\sigma^2} && (23)\\ &= 1-\left(1-\left|\tilde{w}_{i1}^{(k)}\right|^2\right)^{T-1}; \qquad 0\le\left|\tilde{w}_{i1}^{(k)}\right|^2\le 1 && (24) \end{aligned}$$

In order to perform the mapping from γ_k to d_k, the users must know $H_k/(T\sigma^2)$. In this paper we assume that this value is known to the receivers. In a practical system the users must estimate the mean SINR (e.g. by averaging) and then solve (21) for $H_k/(T\sigma^2)$ (e.g. by means of a lookup table).

By construction, the pdf of the decision variable d_k is

$$p(d_k|H_k) = 1; \qquad 0 \le d_k \le 1 \tag{25}$$

which is no function of the instantaneous channel $\mathbf{h}_k$ and is therefore identical for all users. Thus, no user is preferred to the others, independent of their channel quality. In order to find the pdf of the scheduled SNR γ_{PFMS}, we first determine the pdf $p(d_{max}|H_1, H_2, \ldots, H_K)$, where d_{max} is the maximum d for a fixed realization of channel vectors $\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K$.

We have

$$\begin{aligned} p(d_{max}|H_1,H_2,\ldots,H_K) &= \sum_{k=1}^{K} p(d_k, \text{user } k \text{ is scheduled}|H_1,H_2,\ldots,H_K) && (26)\\ &= K\cdot p(d_1, \text{user } 1 \text{ is scheduled}|H_1,H_2,\ldots,H_K) && (27) \end{aligned}$$

because the events ”user k has the maximum d and is scheduled” are mutually exclusive for different users but have the same probability.

Averaging over the channel realizations (in terms of $\mathbf{U}_1, \ldots, \mathbf{U}_K$) we obtain

$$p(d_{max}|H_1, H_2, \ldots, H_K) = K \int\limits_{U_1} \cdots \int\limits_{U_K} p(U_1, \ldots, U_K|H_1, \ldots, H_K) \cdot$$

$$\cdot p(d_1, \text{user } 1 \text{ is scheduled}|U_1, \ldots, U_K, H_1, \ldots, H_K)\, dU_K \ldots dU_1 \tag{28}$$

$$= K \int\limits_{U_1} \cdots \int\limits_{U_K} p(U_1, \ldots, U_K|H_1, \ldots, H_K) \cdot$$

$$\cdot \int\limits_0^{d_{max}} \cdots \int\limits_0^{d_{max}} p(d_1, \ldots, d_K|U_1, \ldots, U_K, H_1, \ldots, H_K)\, dd_2 \ldots dd_K\, dU_K \ldots dU_1 \tag{29}$$

$$= K \int\limits_0^{d_{max}} \cdots \int\limits_0^{d_{max}} \int\limits_{U_1} \cdots \int\limits_{U_K} p(d_1, \ldots, d_K, U_1, \ldots, U_K|H_1, \ldots, H_K)\, dU_K \ldots dU_1\, dd_2 \ldots dd_K$$

$$= K \int\limits_0^{d_{max}} \cdots \int\limits_0^{d_{max}} p(d_1, \ldots, d_K|H_1, \ldots, H_K)\, dd_2 \ldots dd_K \tag{30}$$

$$= K \int\limits_0^{d_{max}} \cdots \int\limits_0^{d_{max}} p(d_1, \ldots, d_K)\, dd_2 \ldots dd_K \tag{31}$$

because the decision variables d_k and the channel powers H_k are statistically independent (cf. (25)).

As $\mathbf{U}_1$ can be constructed to be i.d. distributed, and because $\mathbf{W}$ is a unitary matrix, $\tilde{\mathbf{W}}_1$ conditioned on $\mathbf{W}$ is i.d. distributed, and thus $\tilde{\mathbf{W}}_1$ and $\mathbf{W}$ are independent. This argumentation can be repeated for $\tilde{\mathbf{W}}_2$ to $\tilde{\mathbf{W}}_K$, so we see that $\tilde{\mathbf{W}}_1, \ldots, \tilde{\mathbf{W}}_K$ and therefore also $d_1, \ldots, d_K$ are statistically independent (cf. (24)).

We finally obtain

$$p(d_{max}|H_1, H_2, \ldots, H_K) = K \int\limits_0^{d_{max}} \cdots \int\limits_0^{d_{max}} p(d_1) \cdot \ldots \cdot p(d_K)\, dd_2 \ldots dd_K \tag{32}$$

$$= K d_{max}^{K-1}; \qquad 0 \leq d_{max} \leq 1 \tag{33}$$

Remapping from d to γ, we obtain the pdf of the scheduled SINR, conditioned on the event that user k is scheduled:

$$p(\gamma_{PFMS}|H_k, \text{user } k \text{ is scheduled}) =$$

$$\frac{K(T-1)}{H_k} \left[1 - \left(1 - \frac{H_k + T\sigma^2}{H_k(1+\gamma)} \gamma \right)^{T-1} \right]^{K-1}$$

$$\cdot \left(1 - \frac{H_k + T\sigma^2}{H_k(1+\gamma)} \gamma \right)^{T-2} \frac{H_k + T\sigma^2}{(1+\gamma)^2}; \qquad 0 \leq \gamma_k \leq \frac{H_k}{T\sigma^2} \tag{34}$$

After averaging over the distribution of H_k, we obtain

$$p(\gamma_{PFMS}) = \int_{T\gamma\sigma^2}^{\infty} p(\gamma_{PFMS}|H_k)\frac{1}{(T-1)!}e^{-H}H^{T-1}\,dH \quad (35)$$

$$= \frac{K(T\sigma^2)^T}{(T-2)!}\sum_{i=0}^{K-1}\binom{K-1}{i} \cdot \sum_{j=0}^{i(T-1)+T-2}\binom{(i+1)T-i-2}{j}(-1)^{i+j}\frac{1}{(1+\gamma)^{j+2}} \cdot \sum_{l=0}^{j+1}\binom{j+1}{l}\gamma^{T+j-l}E_l(T\sigma^2\gamma) \quad (36)$$

where E_l is the exponential integral [1] defined as

$$E_l(z) = \int_1^{\infty}\frac{e^{-zt}}{t^l}\,dt \quad (37)$$

Eq. (22) defines one possibility to map the SINR to a decision variable that is identically distributed for all users. The question arises whether the performance of the system depends on the choice of the mapping. At least if the mapping is monotonously increasing with γ, the answer is no according to the following proposition:

Proposition 1: If $d = f(\gamma, H)$ is a monotonously increasing function in γ, so that $p(d|H)$ is identical for all users (i.e. no function of H), then the pdf of the scheduled SINR after mapping d back to γ does not depend on the actual choice of the mapping function f.

Proof: We have

$$d = f(\gamma, H) \quad (38)$$

so the pdf of the maximum decision variable of K users is

$$p(d_{max}) = K\left(F(d)\right)^{K-1}p(d) \quad (39)$$

where $F(d)$ is the cumulative distribution function of d.

A change of variables ($\gamma = f^{-1}(d,H) =: g(d,H)$) leads to

$$p(\gamma_{pfs}|H) = K\left(F_d(f(\gamma,H))\right)^{K-1}p_d(f(\gamma,H))\left|\frac{\partial}{\partial\gamma}f(\gamma,H)\right| \quad (40)$$

$$= K\left[\int_{g(-\infty,H)}^{\gamma}p_d(f(\gamma',H))\frac{\partial}{\partial\gamma'}f(\gamma',H)\,d\gamma'\right]^{K-1}p_d(f(\gamma,H))\left|\frac{\partial}{\partial\gamma}f(\gamma,H)\right|$$

$$= K\left[\int_{g(-\infty,H)}^{\gamma}p_d(f(\gamma',H))\left|\frac{\partial}{\partial\gamma'}f(\gamma',H)\right|\,d\gamma'\right]^{K-1}p_d(f(\gamma,H))\left|\frac{\partial}{\partial\gamma}f(\gamma,H)\right|$$

where the last equation follows from the assumption that $f(\gamma, H)$ is monotonously increasing in γ. So we end up with

$$p(\gamma_{pfs}|H) = K\left[\int_{g(-\infty,H)}^{\gamma}p(\gamma'|H)\,d\gamma'\right]^{K-1}p(\gamma|H) \quad (41)$$

$$= K\left[F(\gamma|H)\right]^{K-1}p(\gamma|H) \quad (42)$$

which depends only on the conditional distribution of the SINR, but not on the mapping function $f(\gamma, H)$. This concludes the proof. ■

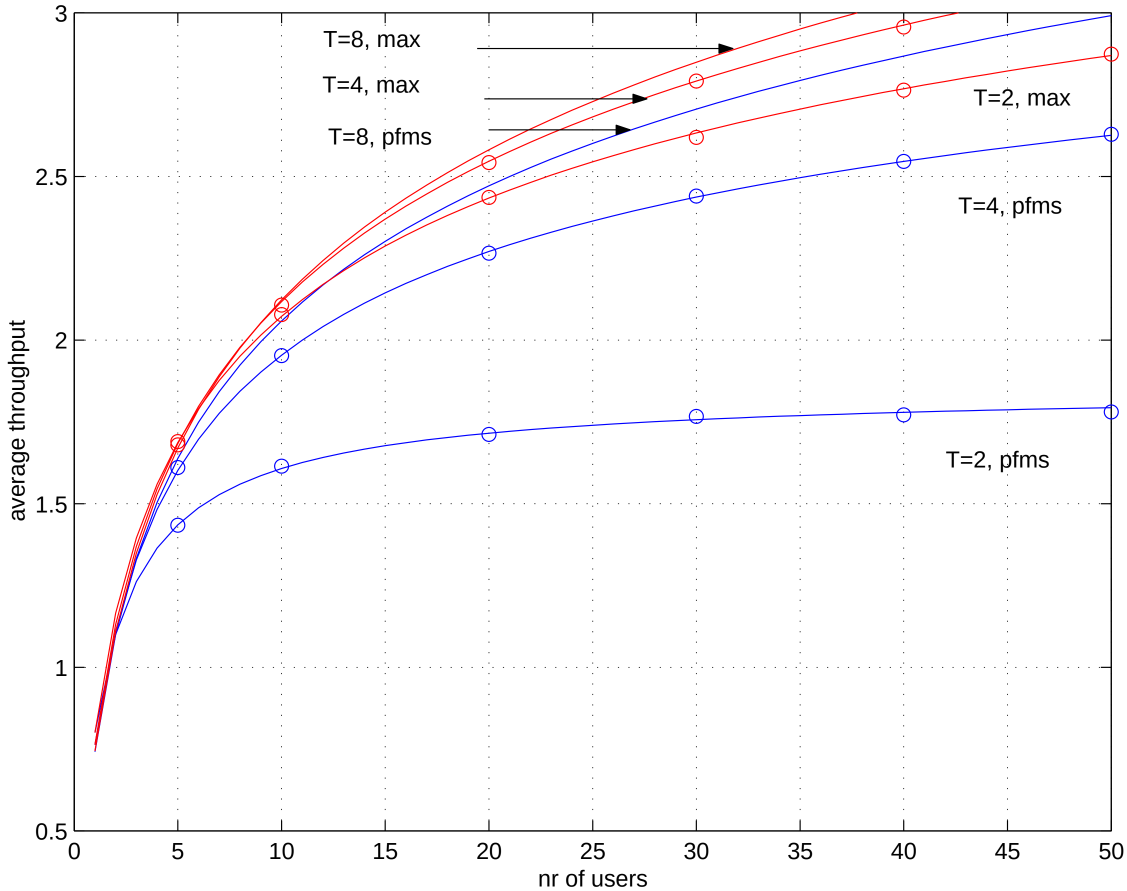

Fig. 2. Average throughput in b/s/Hz averaged over slow Rayleigh fading at 0 dB SNR with the proportional fair multibeam scheduling algorithm (pfms) and the max SINR scheduler for 2, 4, and 8 antennas. The circles show the simulated values, the dashed lines denote the beamforming throughput R^{bf}.

VI. Numerical Results

The average throughput of each beam is calculated by[2]

$$E(R) = T \int_0^\infty \log_2(1+\gamma)p(\gamma)\, d\gamma \tag{43}$$

which can be evaluated by numerical integration for any number of antennas and users, and which is plotted in Fig. 2 for both the proportional fair multibeam scheduler and the max SINR scheduler along with simulated values for a scenario where all users have a mean SNR of 0 dB ($\sigma^2 = 1$).

We notice that the loss of the PFMS scheduler compared with the max SINR scheduler becomes smaller as the number of antenna elements increases. Similar results can be obtained for the single beam case [5]. The same phenomenon but from another perspective was reported in [7]. Here, the authors examined the max SINR scheduler and noticed that it becomes fairer as the number of antennas increases.

In order to see the increase in throughput for a multibeam system compared with a single beam system, we repeat the results from [5] for a single beam system in Fig. 3.

[2]This is the ergodic capacity. Other measures can easily be obtained if the functional relationship between these measures and the SINR is known.

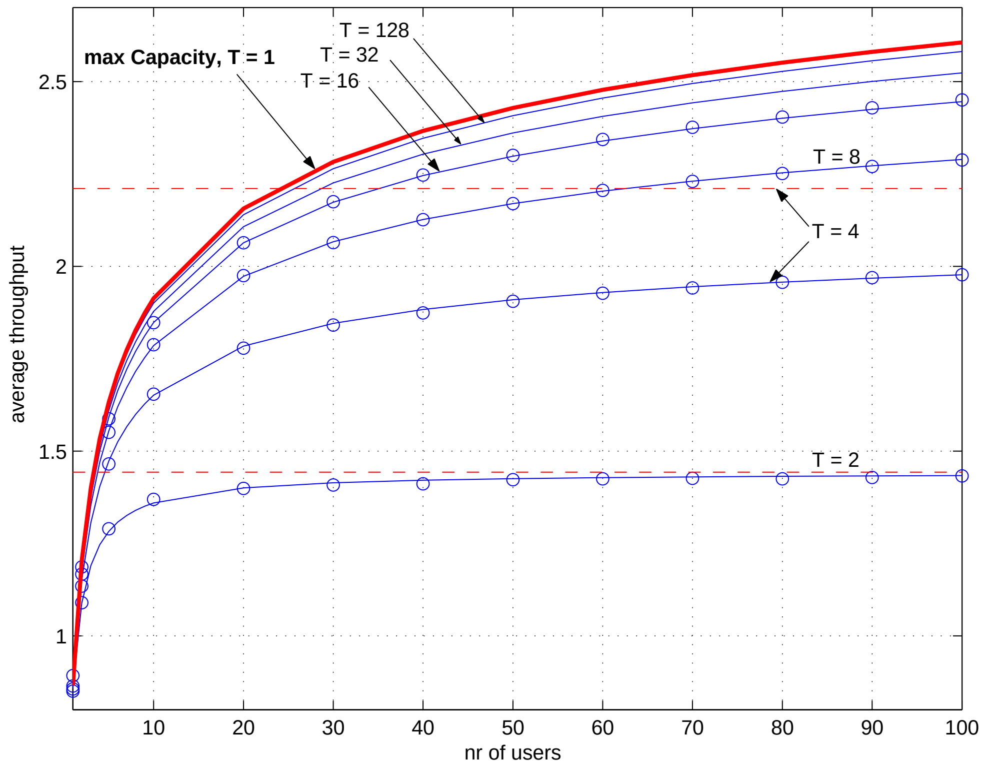

Fig. 3. Average throughput in b/s/Hz averaged over slow Rayleigh fading at 0 dB SNR with the proportional fair single beam scheduling algorithm. The circles show the simulated values, the dashed lines denote the beamforming throughput R^{bf}.

VII. Asymptotic behavior of the PFMS scheduler for large K

In this section we analyze the properties of the PFMS scheduler as the number of users approaches infinity. We use (34) to express the mean throughput:

$$\begin{aligned}
&E(R_{PFMS})\\
&= T\int_0^\infty \int_0^{\frac{H_k}{T\sigma^2}} \log_2(1+\gamma)\frac{K(T-1)}{H_k}\left(1-\left(1-\frac{H_k+T\sigma^2}{H_k(1+\gamma)}\gamma\right)^{T-1}\right)^{K-1}\\
&\quad\cdot\left(1-\frac{H_k+T\sigma^2}{H_k(1+\gamma)}\gamma\right)^{T-2}\frac{H_k+T\sigma^2}{(1+\gamma)^2}d\gamma\frac{1}{(T-1)!}e^{-H_k}H_k^{T-1}\,dH_k\\
&= T\int_0^\infty\left[\log_2(1+\frac{H_k}{T\sigma^2})-\log_2(e)\int_0^1\frac{H_k}{H_k+T\sigma^2-H_kz}\left(1-(1-z)^{T-1}\right)^K dz\right]\\
&\quad\cdot\frac{1}{(T-1)!}e^{-H_k}H_k^{T-1}\,dH_k \qquad (44)
\end{aligned}$$

Thus

$$\begin{aligned}
&-\log_2(e)\int_0^1\left(1-(1-z)^{T-1}\right)^K dz\\
&\quad\cdot\int_0^\infty\frac{H_k}{\sigma^2(T-1)!}e^{-H_k}H_k^{T-1}\,dH_k\\
&= -\frac{T}{\sigma^2}\log_2(e)\int_0^1\left(1-(1-z)^{T-1}\right)^K dz \qquad (45)\\
&\leq E(R_{PFMS})-T\int_0^\infty\frac{\log_2(1+H_k/T\sigma^2)}{(T-1)!}e^{-H_k}H_k^{T-1}\,dH_k\leq 0 \qquad (46)
\end{aligned}$$

As the integral in (45) vanishes for $K \to \infty$, we have

$$\begin{aligned} \lim_{K\to\infty} KR^{(K)} &= \lim_{K\to\infty} E(R_{PFMS}) \\ &= T\int_0^\infty \frac{\log_2(1+H/T\sigma^2)}{(T-1)!} e^{-H} H^{T-1}\, dH \\ &= T\int_0^\infty \log_2(1+SINR_{bf}) p(H)\, dH \\ &= R^{bf} \end{aligned}$$

where $R^{(K)}$ is the average throughput a user achieves in the opportunistic multibeam beamforming system when there are K users in the cell, and R^{bf} is the throughput that a user achieves when it is in the coherent beamforming constellation *for all beams but with no interference between beams*, i.e. when the instantaneous SINR on all T beams is $H/T\sigma^2$.

The first equation follows from the fact that the probability that a user is scheduled is 1/K (all users have identically distributed d_k), which completes the proof that each user's throughput approaches the throughput achieved by coherent beamforming and round robin scheduling if K goes to infinity.

VIII. Conclusion

In order to increase the spectral efficiency of cellular wireless systems, all available forms of diversity should be exploited. One novel concept to enhance the exploitation of multiuser diversity is random beamforming and channel aware scheduling as proposed in [8], where the signal is multiplied by a time varying weight vector to increase the natural dynamic of the channel and a scheduler in the basestation determines the user to transmit to based on the instantaneous SNR fed back from the users. This diversity can be further increased if the base station transmits multiple data streams at the same time potentially to different users, because with this technique the probability that a user has a good channel quality can be increased.

In this paper, we analyzed two schedulers, the max SINR scheduler which maximizes the total average throughput by always transmitting to the users who have the largest SINR, and a novel proportional fair multibeam scheduler which guarantees that all users are scheduled the same amount of time. We derived the pdf of the scheduled SINR, calculated the average throughput by numerical integration and confirmed the analytical results by means of simulation.

Finally, we examined the asymptotic behavior of the proportional fair multibeam scheduler and showed that it approaches coherent beamforming with round robin scheduling if there are many users in the cell.

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