The Effect of Imperfect SNR Knowledge on Multiantenna Multiuser Systems with Channel Aware Scheduling

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***Abstract*— We analyze a cellular communication system in which a basestation (BTS) or access point transmits packet data to several mobile data users by means of a TDMA scheme. All users estimate their instantaneous signal-to-noise-ratio (SNR) in each slot and feed this information back to the BTS. A scheduler in the BTS then uses this information to allocate the channel resource to the user which maximizes a certain metric. We are interested in assessing the sensitivity of the system performance in terms of spectral efficiency per cell with respect to an imperfect knowledge of the multiuser channel, expressed by the estimated SNR of all users. By assuming a block fading channel model for each user, data-aided maximum-likelihood intra-slot SNR estimation can be performed if known pilot symbols are transmitted in each slot. We derive a novel SNR estimator for the block fading channel, which takes the channel statistics into account. The new estimator clearly outperforms the AWGN ML estimator in terms of system performance. Not only is the spectral efficiency larger for the system, but the optimum spectral efficiency is also achieved with fewer pilot symbols per slot.**

I. Introduction

Wireless communications are currently facing a tremendous boost on the advent of the worldwide introduction of third generation cellular and next-generation wireless LAN standards. Both the amount of internet traffic and cellular wireless communications have grown strongly in the past few years, creating a huge market for high speed wireless access. As wireless data traffic is forecasted to be increasingly asymmetrical, the downlink of future systems will constitute the bottleneck and requires optimization. This can only be achieved by systematically utilizing all available forms of diversity. Presently, diversity forms such as spatial, multipath or code diversity aim at either increasing the quality of a single communication link, or increasing the number of terminals that can communicate with, say, a basestation. In an environment where one transmitter communicates with many mobile receivers, another form of diversity can be exploited in order to improve spectral efficiency: *multiuser diversity*. If the transmitter has knowledge about each receiver's channel quality and a packet-based transmission protocol is used, a ”smart” channel aware scheduling algorithm transmits data packets only to users with ”good” channel conditions. With growing number of users in the system, the probability that at least one user has good channel conditions increases and thus the overall throughput can be significantly improved. The gain obtained by scheduling users in such a way is large when fading for all users is fast and has a high dynamic range. In picocells with typically slow fading or in macrocells with little scattering around the transmitter, it can be shown that employing multiple antennas along with random beamforming increases multiuser diversity by ”randomizing” the channels [1]. This is a completely new design paradigm, which is in contrast to conventional approaches where one attempts to exploit diversity in order to average over the fading process.

A number of challenging problems arise if one is interested in exploiting multiuser diversity in a real system. One common assumption is that the transmitter has knowledge about the channel quality of each user, upon which the scheduler decides which user to schedule. This requires either estimates from uplink transmission which are only available in TDD systems, or feedback from the users to the BTS. We will assume that each user estimates its signal-to-noise ratio (SNR) and feeds this estimate back to the BTS. In this paper, we analyze the effect of estimation errors on system throughput in a system where the SNR is used both to schedule the users and to select a transmission mode in terms of modulation scheme and code rate. We derive a novel SNR estimator for the instantaneous SNR in block fading channels, which is based on the maximum likelihood (ML) SNR estimator for the AWGN channel and uses knowledge about the channel statistics.

The outline of the paper is as follows. In section II, the signal and channel models are presented. The novel SNR estimator is derived in section III. In section IV we assess the sensitivity of a cellular multiuser system with channel-aware scheduling to estimation errors in the decision variable, before a conclusion is presented in section V.

II. System Model

Consider a multiantenna basestation (BTS) serving a packet-based downlink of a cell in a cellular wireless communication system. We assume that a packet scheduler decides on the user to be served at the beginning of each slot. Each user estimates its instantaneous SNR in each slot and feeds the estimate back to the BTS. There, random beamforming (termed ”opportunistic beamforming” in [1]) is used to increase the channel dynamic, which can be shown to be beneficial for system throughput. We are interested in assessing the sensitivity of the system performance in terms of overall cell throughput with respect to an imperfect knowledge of the SNR. We assume the availability of pilot symbols being inserted periodically into the transmitted data stream, enabling data-aided channel estimation for coherent detection while the user actually receives data. The users make use of these available pilot symbols to estimate the SNR at all times, i.e. also when they are not scheduled to receive data.

Our system comprises a basestation with T antennas, communicating with U users with one antenna each via a TDMA scheme and a fixed slot length of N symbols, N_P of which are pilot symbols and $N_D = N - N_P$ are data symbols. We assume a flat-fading channel for all users. This is a justified simplification if we use OFDM for frequency-selective fading channels and assume the equivalent narrowband channel on each OFDM-subcarrier to be flat. We shall focus on the case when the inherent dynamic of the channel, determined by the motion of the user, is slow, and we employ slot-wise random beamforming with uncorrelated complex-valued antenna weights as proposed in [1]. The resulting channel is then block fading. The received symbol sequence at user u is given by

$$r_k^{(u)} = c_l^{(u)}\, a_k^{(u)} + n_k^{(u)} \tag{1}$$

where $c_l^{(u)}$ is the effective channel coefficient seen by user u with $l = \lfloor k/N \rfloor$ being the slot index, the $a_k^{(u)}$ are the transmitted symbols and the $n_k^{(u)}$ are samples from a zero-mean white Gaussian noise process. We do not consider time-varying intercell interference in the scope of this paper, so n_k models only thermal noise in the receiver. The effective channel in each slot is given by

$$c_l^{(u)} = \mathbf{w}_l^H\, \mathbf{c}_l^{(u)} \tag{2}$$

where $\mathbf{w}_l \in \mathrm{C}^{T\times 1}$ is the antenna weight vector in slot l, normalized such that the sum of powers over the antennas is one ($\mathbf{w}_l^H \mathbf{w}_l = 1$) and $\mathbf{c}_l^{(u)}$ is the channel vector for user u in slot l. We assume rich scattering around the BTS, such that the elements of the physical channel vector $\mathbf{c}^{(u)}$ are uncorrelated complex Gaussian random variables. It can be shown that the effective channel coefficient $c^{(u)}$ is also complex Gaussian distributed with the same variance as the physical channel if $\mathbf{w}_l^H \mathbf{w}_l = 1$ [6].

III. Estimation of the SNR

A. Maximum Likelihood SNR Estimation

We will focus on one user for the derivation of the estimation algorithms. The channel model from (1) can be rewritten as

$$r_k = \sqrt{\mathcal{P}_l}\, e^{j\phi_l}\, a_k + \sqrt{\mathcal{N}_l}\, \tilde{n}_k \tag{3}$$

where $\mathcal{P}_l$ and $\mathcal{N}_l$ denote the signal and noise power, respectively, ϕ_l is the channel phase and the $\tilde{n}_k$ are samples from a zero-mean, complex valued Gaussian noise process, with real and imaginary parts having variance $1/2$. We are interested in estimating the ratio $\gamma_l = \mathcal{P}_l/\mathcal{N}_l$ in each slot, and with the block fading assumption we can perform intra-slot SNR estimation using the ML estimator derived for the AWGN channel. The derivation of the ML estimator for the modified AWGN channel follows along the lines of [7]. The details (see e.g. [5]) are omitted here. The resulting estimator is given by

$$\hat{\gamma} = \frac{\hat{\mathcal{P}}}{\hat{\mathcal{N}}} = \frac{|\mathbf{a}^H\,\mathbf{r}|^2}{N_P\,\mathbf{r}^H\,\mathbf{r} - |\mathbf{a}^H\,\mathbf{r}|^2} \tag{4}$$

where $\mathbf{a} = [a_0 \dots a_{N_P-1}]^T$ is the vector of transmitted pilot symbols and $\mathbf{r} = [r_0 \dots r_{N_P-1}]^T$ is the vector of received pilot symbols, both in the respective slot. The probability density function of the ML SNR estimator, conditioned on the true instantaneous SNR γ, is given by [5]

$$p_{\hat{\gamma}}(\hat{\gamma}\,|\,\gamma) = e^{-N_P\gamma}\frac{N_P-1}{(1+\hat{\gamma})^{N_P}}\;{}_1F_1\left(N_P, 1; \frac{N_P\,\gamma\,\hat{\gamma}}{1+\hat{\gamma}}\right) \tag{5}$$

where ${}_1F_1\,(a,b;z)$ is the confluent hypergeometric function, defined by

$${}_1F_1\,(a,b;z) = \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\,\Gamma(b)}{\Gamma(a)\,\Gamma(b+n)}\frac{z^n}{n!} \tag{6}$$

The expected value and variance of $\hat{\gamma}$, conditioned on γ, are given by [5]

$$E[\hat{\gamma}\,|\,\gamma] \;=\; \frac{N_P}{N_P-2}\;\left(\gamma + \frac{1}{N_P}\right) \tag{7}$$

$$var[\hat{\gamma}\,|\,\gamma] \;=\; \frac{1}{(N_P-2)(N_P-3)}\left(\frac{(N_P\gamma+1)^2}{N_P-2} + 2N_P\gamma + 1\right) \tag{8}$$

The ML estimator of the SNR is obviously biased. However, the bias only depends on the length of the estimator N_P, is thus known in the receiver and can be compensated for. An unbiased version of the ML estimator (termed UML) is then given by

$$\hat{\gamma}_{UML} = \frac{N_P-2}{N_P}\,\hat{\gamma} - \frac{1}{N_P}. \tag{9}$$

It is straightforward to show that the conditional variance of the unbiased estimator is always smaller than that of the ML estimator [5]. It is given by

$$var[\hat{\gamma}_{UML}\,|\,\gamma] = \frac{(N_P\,\gamma)^2 + N_P\,(2\,N_P-2)\,\gamma + N_P - 1}{N_P^2\,(N_P-3)} \tag{10}$$

B. Novel SNR Estimator

The ML SNR estimator is characterized by its conditional pdf (5). If we assume that each receiver knows the mean SNR of its channel, e.g. by averaging, and we furthermore assume Rayleigh fading for each user, resulting in a chi-squared pdf of the SNR, we can incorporate this knowledge and devise a better estimator. To that end, we first compute the conditional pdf of the true instantaneous SNR γ, conditioned on the knowledge of the estimated instantaneous SNR $\hat{\gamma}$ and on the mean SNR $\overline{\gamma}$ of the assumed chi-square distributed channel SNR. With Bayes' rule we have

$$p_\gamma(\gamma\,|\,\hat{\gamma},\overline{\gamma}) = \frac{p_{\hat{\gamma}}(\hat{\gamma}\,|\,\gamma,\overline{\gamma})\;\; p_\gamma(\gamma\,|\,\overline{\gamma})}{p_{\hat{\gamma}}(\hat{\gamma}\,|\,\overline{\gamma})} \tag{11}$$

where $p_{\hat{\gamma}}(\hat{\gamma}\,|\,\gamma,\overline{\gamma}) = p_{\hat{\gamma}}(\hat{\gamma}\,|\,\gamma)$ is the conditional pdf of the ML estimator, $p_\gamma(\gamma\,|\,\overline{\gamma})$ is the pdf of the channel SNR and the denominator is the integral of the numerator over γ. A novel estimator, termed EML, is then given by computing the expected value of (11) with respect to γ:

$$\begin{aligned} E[\gamma\,|\,\hat{\gamma},\overline{\gamma}] &= \int_0^\infty p_\gamma(\gamma\,|\,\hat{\gamma},\overline{\gamma})\,\gamma\,d\gamma \\ &= \frac{\overline{\gamma}}{1+N_P\overline{\gamma}}\left(1+\frac{N_P^2\,\overline{\gamma}\hat{\gamma}}{1+N_P\,\overline{\gamma}+\hat{\gamma}}\right) \end{aligned} \tag{12}$$

The compact closed-form result (12) is derived in the appendix. The novel estimator can be interpreted as a postprocessing of standard ML SNR estimates, with additional side information about the mean channel SNR, as illustrated in Figure 1.

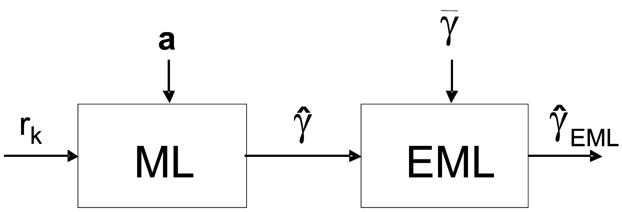

Fig. 1. Novel SNR estimator: postprocessing ML estimates

IV. System Sensitivity to Estimated SNR

We consider two different multiuser scenarios and assess the system sensitivity to imperfect SNR knowledge. The first scenario is a cell with the users having equal mean SNR of 0 dB. The scheduler performs maximum carrier-to-interference scheduling (max C/I), i.e. it schedules the user with the best instantaneous SNR:

$$u_{max;l} = \arg\max_u \left(\gamma_l^{(u)}\right), \qquad u \in [1 \ldots U] \tag{13}$$

with $\gamma_l^{(u)}$ being the SNR of user u in slot l. In the second scenario, the users are uniformly distributed in a disc around the BTS, with the mean SNR of each user determined by the free space path loss. The cell geometry is chosen such that the mean SNRs of the users are between 0 and 20 dB. The scheduler uses proportional fair scheduling (PFS) [1], where the metric to be maximized is the instantaneous supportable rate $R_l^{(u)}$, which is an increasing function of the SNR, normalized to the average received rate in the past $T_l^{(u)}$:

$$u_{max;l} = \arg\max_u \left(\frac{R_l^{(u)}}{T_l^{(u)}}\right) \tag{14}$$

Weaker users are thus also served at the expense of system throughput, indicating the fundamental throughput-fairness tradeoff in scheduling. The average rate for each user can be updated according to

$$T_{l+1}^{(u)} = \begin{cases} (1-1/t_c)\,T_l^{(u)} + R_l^{(u)}/t_c, & u_{max;l} = u \\ (1-1/t_c)\,T_l^{(u)}, & otherwise \end{cases} \tag{15}$$

where t_c represents the memory of the rate averaging for each user, which can be adapted to higher-layer requirements, e.g. maximum cell delay. For the mapping of the SNR to a supportable datarate, we performed link level simulations[1] with a system using adaptive modulation and coding and a simple ARQ protocol, which requests a retransmission of a packet (slot) when at least one bit error occurs. The results are shown in Figure 2. The average spectral efficiency, measured in the number of transmitted bits per channel use, is plotted over the SNR for different modulation schemes and coderates. These transmission modes were taken from the IEEE 802.11a standard [8].

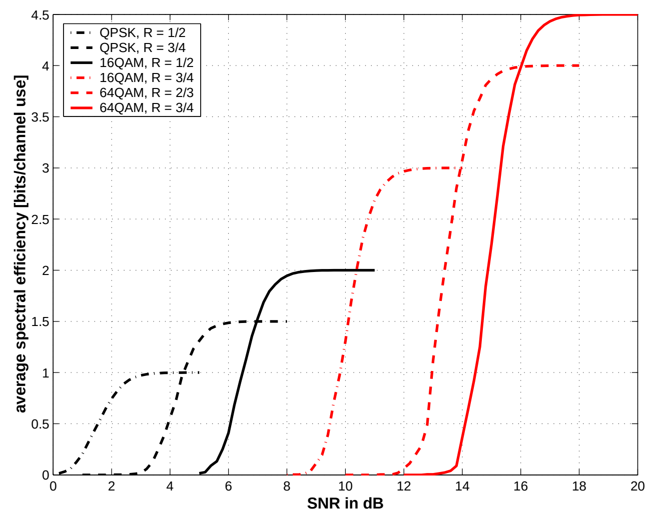

Fig. 2. Link level simulation results: average bits per channel use

The simulation flow for the system simulation is then as follows:

1. In each slot, each user estimates its instantaneous SNR with one of the proposed algorithms ML, UML or EML.
2. The scheduler schedules the user for the current slot according to the estimated SNR.
3. For the scheduled user, the transmission mode is chosen according to the estimated SNR and the actual spectral efficiency is determined by the number of bits per channel use of the chosen transmission mode at the true instantaneous SNR.

[1]All simulations were performed with Synopsys CoCentric System Studio

One interesting tradeoff with data-aided estimation is estimation accuracy vs. spectral efficiency [2]. To that end, the results from Figure 2 are multiplied by a factor representing the loss in spectral efficiency due to the transmission of pilot symbols, common in data-aided estimation [3]:

$$R_l^{(u)} = \eta_l^{(u)} \frac{N_D}{N_D + N_P} \tag{16}$$

where $\eta_l^{(u)}$ is the spectral efficiency of the chosen transmission mode for user u at its true instantaneous SNR and the second factor is the ratio between the information symbols per slot and the total number of symbols per slot. The measure of interest in then the average spectral efficiency of the cell, which is given by the sum of the average spectral efficiencies of the users: $R_{system} = \sum_{n=1}^{U} E_l[R_l^{(u)}]$. Here, E_l denotes expectation with respect to time (l is the slot index). In the system simulations for scenario 2, we average additionally over several user (and thus mean SNR) distributions. The impact of imperfect SNR knowledge on system performance is shown in Figure 3 for scenario 2 with $t_c = 100$. For each of the SNR estimation algorithms and for perfect SNR knowledge (the rightmost two bars), the average spectral efficiency of the cell is plotted for 10 and 100 users in the cell, for a fixed total slot length of $N = 128$ symbols and for estimator lengths $N_P = 8$ and $N_P = 16$. For the standard ML estimator and its unbiased version, the spectral efficiency for $N_P = 8$ and 100 users is smaller than for 10 users, i.e. no multiuser diversity can be exploited due to poor SNR estimation. This is not the case for the new EML estimator. It is seen that the spectral efficiency is significantly larger with EML estimation and that the losses compared to perfect SNR knowledge are significantly smaller than with the two other estimators.

The tradeoff between estimation accuracy and spectral efficiency is shown in Figure 4 for scenarios 1 and 2 with 100 users. The average spectral efficiency for the system R_{system} is plotted over the number of pilot symbols per slot N_P for $N = 128$. Clearly, the expected rate for perfect SNR knowledge (the solid curves) is a linearly decreasing function of N_P. For a small number of pilots, the rates increase for all estimators with increasing N_P due to increased estimation accuracy. For large N_P, the rates decrease due to the increasingly unfavourable ratio of pilot to data symbols. The optimum number of pilot symbols is $N_P = 15\ldots30$. Three observations are noteworthy:

1. the novel EML estimator yields the largest average spectral efficiency in both scenarios
2. its optimum number of pilots is smaller than that of the other two estimators
3. the degradation in spectral efficiency is less severe when the number of pilots is suboptimal

Similar results have been obtained for other simulation parameterizations. It has also been verified that the novel

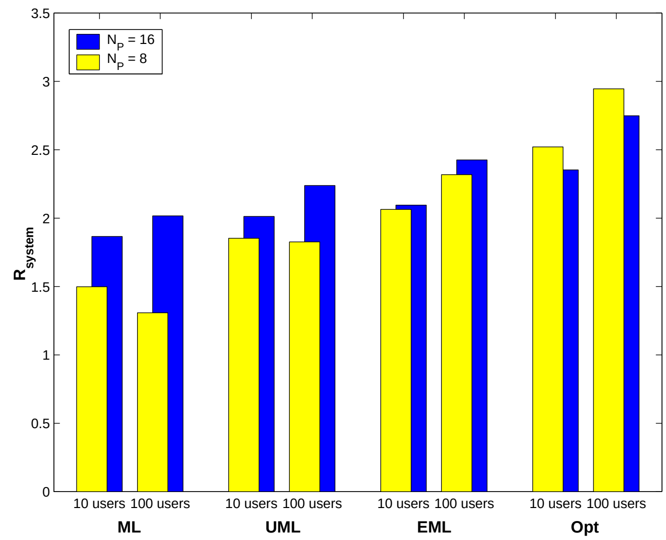

Fig. 3. Average spectral efficiency for scenario 2

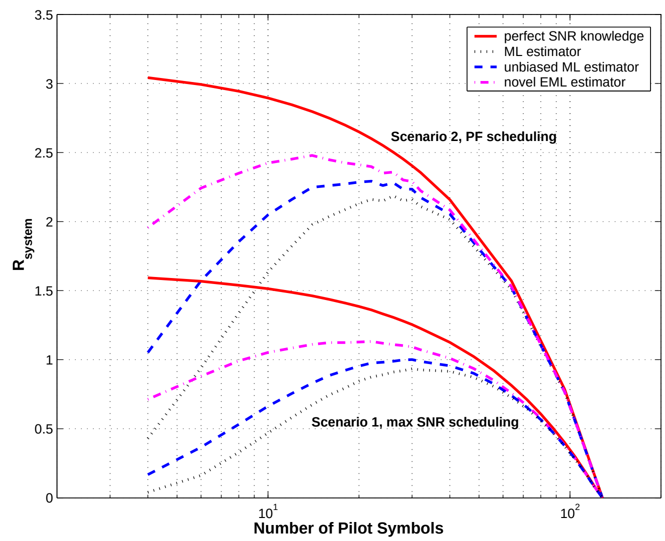

Fig. 4. Average spectral efficiency

estimator is not sensitive to a mismatch of the assumption about the channel statistics, i.e. if the channel amplitude distribution is Rice or Nakagami instead of the assumed Rayleigh distribution.

V. Conclusion

A novel data-aided estimator for the signal-to-noise ratio in block fading channel models was derived in this paper, based on the maximum likelihood SNR estimator for the AWGN channel. Its superior estimation performance was demonstrated in multiuser cellular scenarios with packet-based transmission, adaptive modulation and coding, ARQ and channel aware scheduling. It was shown that the new estimator not only yields a larger system spectral efficiency, but that the maximum spectral efficiency is achieved with less pilot symbols per slot.

Further investigations are under way to devise SNR estimation algorithms for scenarios where the block fading is no longer uncorrelated. Also, the system performance and sensitivity is being analyzed if ARQ with incremental redundancy instead of adaptive modulation and coding is used.

APPENDIX

In this section we derive the novel estimator for the instantaneous SNR. Using Bayes' rule, we can express the pdf of the true instantaneous SNR γ, conditioned on the estimated instantaneous SNR $\hat{\gamma}$ and the mean SNR of the channel $\overline{\gamma}$, which we will assume to be known, i.e. through averaging of the channel, as

$$p_\gamma(\gamma\,|\,\hat{\gamma},\overline{\gamma}) = \frac{p_{\hat{\gamma}}(\hat{\gamma}\,|\,\gamma)\, p_\gamma(\gamma\,|\,\overline{\gamma})}{p_{\hat{\gamma}}(\hat{\gamma}\,|\,\overline{\gamma})} \qquad (17)$$

We compute the denominator first. It is given by

$$\begin{aligned} p_{\hat{\gamma}}(\hat{\gamma}\,|\,\overline{\gamma}) &= \int_0^\infty p_{\hat{\gamma}}(\hat{\gamma}\,|\,\gamma)\, p_\gamma(\gamma\,|\,\overline{\gamma})\, d\gamma \\ &= \frac{N_P-1}{\overline{\gamma}(1+\hat{\gamma})^{N_P}} \int_0^\infty e^{-\gamma\left(N_P+\frac{1}{\overline{\gamma}}\right)} \\ &\quad \cdot\ {}_1F_1\left(N_P, 1; \frac{N_P\,\gamma\,\hat{\gamma}}{1+\hat{\gamma}}\right) d\gamma \qquad (18) \\ &= \frac{N_P-1}{\overline{\gamma}(1+\hat{\gamma})^{N_P}} \sum_{n=0}^\infty \left(\frac{N_P\,\hat{\gamma}}{1+\hat{\gamma}}\right)^n \frac{\Gamma(N_P+n)}{\Gamma(n+1)\, n!} \\ &\quad \cdot \int_0^\infty e^{-\gamma\left(N_P+\frac{1}{\overline{\gamma}}\right)} \gamma^n\, d\gamma \qquad (19) \\ &= \frac{N_P-1}{\overline{\gamma}(1+\hat{\gamma})^{N_P}\,(N_P+\frac{1}{\overline{\gamma}})} \sum_{n=0}^\infty \frac{(N_P+n-1)!}{(N_P-1)!\, n!} \\ &\quad \cdot \left(\frac{N_P\,\hat{\gamma}}{(1+\hat{\gamma})(N+\frac{1}{\overline{\gamma}})}\right)^n \qquad (20) \\ &= \frac{N_P-1}{\overline{\gamma}} \frac{\left(N_P+\frac{1}{\overline{\gamma}}\right)^{N_P-1}}{\left((1+\hat{\gamma})(N_P+\frac{1}{\overline{\gamma}})-N_P\,\hat{\gamma}\right)^{N_P}} \qquad (21) \end{aligned}$$

where for (19) we use the series definition of the confluent hypergeometric function (6), (20) follows from

$$\int_0^\infty \gamma^n\, e^{-\gamma\left(N_P+\frac{1}{\overline{\gamma}}\right)}\ d\gamma = \frac{n!}{(N_P+\frac{1}{\overline{\gamma}})^{n+1}} \qquad (22)$$

which is computed by repeated integration by parts, and for (21) we use the binomial series with negative exponent [9]:

$$\sum_{n=0}^\infty \frac{(N_P+n-1)!}{(N_P-1)!\ n!} A^n = (1-A)^{-N_P} \qquad N_P > 0,\ |A| < 1 \quad (23)$$

It is easily verified that $|A| < 1$ with $A = N_P\,\hat{\gamma}/((1+\hat{\gamma})(N+1/\overline{\gamma}))$ from equation (20) is always valid for $\hat{\gamma} > 0$ and $\overline{\gamma} > 0$. Inserting (21) into (17) we have, after some algebra,

$$\begin{aligned} p_\gamma(\gamma\,|\,\hat{\gamma},\overline{\gamma}) &= \frac{1+N_P\overline{\gamma}}{\overline{\gamma}} \left(\frac{1+N_P\,\overline{\gamma}\,\hat{\gamma}}{(1+N_P\,\overline{\gamma})\,(1+\hat{\gamma})}\right)^{N_P} \\ &\quad \cdot e^{-\gamma\left(N_P+\frac{1}{\overline{\gamma}}\right)}\ {}_1F_1\left(N_P, 1; \frac{N_P\,\gamma\,\hat{\gamma}}{1+\hat{\gamma}}\right) \qquad (24) \end{aligned}$$

The expected value of the γ, conditioned on $\hat{\gamma}$ and $\overline{\gamma}$, is then computed by integrating (24) over γ:

$$\begin{aligned} E[\gamma\,|\,\hat{\gamma},\overline{\gamma}] &= \int_0^\infty \gamma \cdot p_\gamma(\gamma\,|\,\hat{\gamma},\overline{\gamma})\, d\gamma \\ &= \frac{1+N_P\overline{\gamma}}{\overline{\gamma}} \left(\frac{1+N_P\,\overline{\gamma}+\hat{\gamma}}{(1+N_P\overline{\gamma})(1+\hat{\gamma})}\right)^{N_P} \\ &\quad \cdot \int_0^\infty \gamma\, e^{-\gamma(N_P+\frac{1}{\overline{\gamma}})}\ {}_1F_1\left(N_P, 1; \frac{N_P\,\gamma\,\hat{\gamma}}{1+\hat{\gamma}}\right) d\gamma \quad (25) \end{aligned}$$

Using similar techniques as for (19)-(21) we finally get to the result

$$E[\gamma\,|\,\hat{\gamma},\overline{\gamma}] = \frac{\overline{\gamma}}{1+N_P\overline{\gamma}} \left(1 + \frac{N_P^2\,\overline{\gamma}\,\hat{\gamma}}{1+N_P\,\overline{\gamma}+\hat{\gamma}}\right) \qquad (26)$$

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