Maximum Ratio Combining of Correlated Diversity Branches with Imperfect Channel State Information and Colored Noise

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Abstract— Recently, an analytical expression for the mean bit error probability of a binary modulated signal has been derived for a receiver performing maximum ratio combining of correlated diversity branches based on imperfect channel state information [1].
In this work, these results are extended to the general case of colored additive noise. Furthermore, an alternative solution is derived which yields numerically stable results also in the case of the respective channel and interference covariance matrices having closely-spaced or multi-fold eigenvalues.
Finally, the results are applied to a spatio-temporal W-CDMA Rake receiver, where the effects of multiple-access-interference (MAI) and interpath interference (IPI), due to multipath propagation, on the mean bit error probability are investigated.

I. INTRODUCTION

Maximum ratio combining (MRC) is a special form of general diversity combining, by which multiple replicas of the same information-bearing signal received over different diversity branches are combined so as to maximize the instantaneous SNR at the combiner output [2]. It is a classical and powerful technique to mitigate the effects of severe fading, which occurs particularly in wireless communication systems. However, in many applications like in CDMA systems, the channel fading coefficients are in general mutually correlated and have different second order statistics affecting the performance of MRC. Examples for this are closely spaced antenna elements at the receiver and a multipath intensity profile with unequal channel tap powers, respectively. In addition, the channel coefficients are not known at the receiver and have to be estimated, either by the use of known pilot symbols or in a blind manner. As it is well known this has a significant effect on the performance of MRC, [9]. The significance of imperfect channel knowledge increases with increasing channel dynamics, since the quality of the channel estimation worsens for increasing channel dynamics [3].
There are many contributions dealing with the performance of maximum ratio combining, see e.g. [4]-[9], but neither of them covers the general case of non-identically mutually correlated fading diversity branches with only partial channel state information being available at the receiver. In [10] the analysis has been carried out for the special case of the down-link of a BPSK-based W-CDMA system. Recently, Dietrich and Utschick [1] derived a general analytical expression of the mean bit error probability for BPSK signalling.
In this contribution we extend the result of [1] to also account for the general case that the additive noise of the respective diversity branches is also mutually correlated. Colored interference arises for example in case of a multi antenna element receiver operating in a spatially correlated scenario with multipath propagation or multiple interfering users. In these cases there is a mutual correlation between the noise samples corresponding to the respective antenna element outputs, i.e. the spatial diversity branches.
If the eigenvalues of the channel covariance matrix are closely spaced, the evaluation of the expression for the bit error probability presented in [1] becomes numerically unstable. This happens for example in the case of a spatial Rake receiver employing multiple antenna elements where the spatial correlation between the fading coefficients at the antenna elements is very low. Therefore, we present an alternative numerically stable expression for the mean bit error probability, which implies the numerical evaluation of a one-dimensional integral with a real valued integrand.
This paper is organized as follows. After introducing the system model in Section II, the derivation of the different expressions for the bit error probability is shown in Section III. Finally, in Section IV, the results are applied to a spatio-temporal W-CDMA Rake receiver, where the effect of multiple-access-interference (MAI) and interpath interference (IPI), due to multipath propagation, on the mean bit error probability is investigated. Section V concludes the paper.

II. SIGNAL MODEL

A. Information-Bearing Signal

Assume that BPSK modulated data symbols $a_n^{(d)} \in \{+\sigma_d, -\sigma_d\}$ are transmitted with $\|a_n^{(d)}\|^2 = \sigma_d^2$ and n denoting the time index. Then, the vector $\boldsymbol{x}_n = [x_{0,n}, \ldots, x_{L-1,n}]^T$ comprising the L spatio-temporal diversity branches, which are used for maximum ratio combining at the receiver, may

be generally modelled as

$$\boldsymbol{x}_n = a_n^{(d)}\boldsymbol{h} + \boldsymbol{v}_n, \tag{1}$$

where $\boldsymbol{h} = [h_0, \dots, h_{L-1}]^T$ and $\boldsymbol{v}_n = [v_{0,n}, \dots, v_{L-1,n}]^T$ denote the effective channel vector and the additive noise vector, respectively.

According to the assumption of Rayleigh fading, the effective channel vector $\boldsymbol{h}_n$ and the noise vector $\boldsymbol{v}_n$ are modelled as zero-mean complex Gaussian random vectors with covariance matrix $\boldsymbol{K_h}$ and $\boldsymbol{K_v}$, respectively, i.e.

$$\boldsymbol{h} \sim \mathcal{N}_C(\boldsymbol{0}, \boldsymbol{K_h}) \tag{2}$$
$$\boldsymbol{v}_n \sim \mathcal{N}_C(\boldsymbol{0}, \sigma_v^2 \boldsymbol{K_v}), \tag{3}$$

where σ_v^2 denotes the noise power. Note, that without loss of generality, it can be assumed that the noise and channel covariance matrices are normalized in the way, that it is $\|\text{diag}\{\boldsymbol{K_v}\}\|^2 = 1$ and $\|\text{diag}\{\boldsymbol{K_h}\}\|^2 = 1$, where $\text{diag}\{\cdot\}$ is a vector containing the diagonal elements.

Obviously, $\boldsymbol{x}_n$ is also zero-mean complex Gaussian distributed with covariance matrix

$$\boldsymbol{K_{x_n}} = \mathrm{E}\{\boldsymbol{x}_n\boldsymbol{x}_n^H\} = \sigma_d^2\boldsymbol{K_h} + \sigma_v^2\boldsymbol{K_v}, \tag{4}$$

where it has been assumed, that the noise samples and the effective channel coefficients are independent.

In order to stress the generality of the model in (1), note that $\boldsymbol{x}_n$ may be the result of a linear processing of the R dimensional received signal vector $\boldsymbol{z}_n$, i.e. $\boldsymbol{x}_n = \boldsymbol{W}\boldsymbol{z}_n$ with $\boldsymbol{W}$ being a $(L \times R)$-matrix.

B. Pilot Signal

As in [1], we assume that the receiver does not have perfect channel state information, but that a noisy channel estimate $\widehat{\boldsymbol{h}} = [\hat{h}_0, \dots, \hat{h}_{L-1}]^T$ is available. The channel estimate is obtained by maximum likelihood channel estimation using a block of M pilot symbols $a_m^{(p)} \in \{+\sigma_p, -\sigma_p\}$, with $|a_m^{(p)}|^2 = \sigma_p^2$, and can be expressed as

$$\widehat{\boldsymbol{h}} = \boldsymbol{h} + \frac{1}{M\sigma_p^2}\sum_{m=0}^{M-1} a_m^{(p)*}\boldsymbol{v}_m. \tag{5}$$

The covariance matrix of the channel estimation vector is given by

$$\boldsymbol{K_{\hat{h}}} = \mathrm{E}\{\widehat{\boldsymbol{h}}\widehat{\boldsymbol{h}}^H\} = \boldsymbol{K_h} + \frac{\sigma_v^2}{M\sigma_p^2}\boldsymbol{K_v}, \tag{6}$$

Now, assuming that the additive noise in (1) and the channel estimation noise in (5) are independent, the cross-covariance matrix of $\boldsymbol{x}_n$ and $\widehat{\boldsymbol{h}}$ is given by

$$\boldsymbol{K_{x_n\hat{h}}} = \mathrm{E}\{\boldsymbol{x}_n\widehat{\boldsymbol{h}}^H\} = \sigma_d\boldsymbol{K_h}, \tag{7}$$

where without loss of generality $a_n^{(d)} = +\sigma_d$ has been assumed to be the transmitted symbol.

C. MRC Decision Variable

The decision variable d_n, which is obtained by maximum ratio combining of the diversity branches [4] is given by

$$d_n = \mathrm{Re}\left\{\widehat{\boldsymbol{h}}^H\boldsymbol{x}_n\right\}. \tag{8}$$

III. BIT ERROR PROBABILITY

In this section, the bit error probability P_B, i.e. the probability that it is $\text{sign}(d_n) \neq \text{sign}(a_n)$, is calculated by following the derivation in [1].

By defining

$$\boldsymbol{r}_n = \begin{bmatrix} \widehat{\boldsymbol{h}}_n \\ \boldsymbol{x}_n \end{bmatrix} \tag{9}$$

and

$$\boldsymbol{A} = \frac{1}{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \otimes \boldsymbol{I}_L, \tag{10}$$

it is

$$d_n = \frac{1}{2}\left(\widehat{\boldsymbol{h}}_n^H\boldsymbol{x}_n + \widehat{\boldsymbol{h}}_n^T\boldsymbol{x}_n^*\right) = \boldsymbol{r}_n^H\boldsymbol{A}\boldsymbol{r}_n \tag{11}$$

a hermitian quadratic form. Hence, according to [11], the characteristic function of d_n is given by

$$\Phi_d(\omega) = \prod_{l=0}^{2L-1}\frac{1}{(1 - j\omega\lambda_l)}, \tag{12}$$

where the λ_l are the eigenvalues of $\boldsymbol{A}\boldsymbol{K_r}$ and $\boldsymbol{K_r} = \mathrm{E}\{\boldsymbol{r}_n\boldsymbol{r}_n^H\}$ is the $(2L \times 2L)$ covariance matrix of $\boldsymbol{r}_n$. If all eigenvalues λ_l, $l = 0, \dots, 2L - 1$ are distinct, the pdf of d_n can be calculated via partial fraction expansion and inverse Fourier transform of $\Phi_d(\omega)$ in (12). Subsequently, the bit error probability is obtained from the pdf via integration (see [1]). For distinct eigenvalues, one obtains

$$P_B = \sum_{\substack{l=0 \\ \lambda_l < 0}}^{2L-1}\prod_{\substack{k=0 \\ k \neq l}}^{2L-1}\frac{\lambda_l}{\lambda_l - \lambda_k}. \tag{13}$$

However, if the eigenvalues are not distinct, but multifold eigenvalues exist, it is difficult to evaluate a general closed form solution of the coefficients of the partial fraction expansion. Also, if the eigenvalues get very close to each other, the calculation of the coefficients of the partial fraction expansion, and hence the evaluation of (13), becomes numerically unstable.

An alternative method is to calculate the bit error probability directly via the characteristic function, which results in a numerically stable solution. According to the lemma of Gil-Pelaez (see e.g. [12]), the cumulative density function (cdf) $F_X(x)$ of any random variable X can be calculated via the characteristic function $\Phi_X(\omega)$ of X as follows

$$F_X(x) = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{\mathrm{Im}\left\{\Phi_X(\omega)e^{-jx\omega}\right\}}{\omega}\,d\omega. \tag{14}$$

Noting that the bit error probability is obtained from the cdf of d_n as follows

$$P_B = F_{d_n}\left(d = 0 | a_n = +\sigma_d\right) \tag{15}$$

the bit error probability can be expressed in terms of a single integral

$$P_B = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \mathrm{Im}\left\{\frac{1}{\omega\prod_{l=0}^{2L-1}(1 - j\omega\lambda_l)}\right\}d\omega, \tag{16}$$

which can be evaluated numerically. Note that the integrand is real valued and decays very fast for increasing ω. The limit of the integrand for $\omega \to 0$ exists and can easily be calculated as follows

$$\lim_{\omega \to 0} \operatorname{Im} \left\{ \frac{1}{\omega \prod_{l=0}^{2L-1}(1 - j\omega\lambda_l)} \right\} = \sum_{l=0}^{2L-1} \lambda_l. \quad (17)$$

It remains to give an expression for $\boldsymbol{AK_r}$. Obviously, using (4), (6) and (7) it is

$$\boldsymbol{AK_r} = \frac{1}{2} \left[\begin{array}{cc} \sigma_d \boldsymbol{K_h} & \sigma_d \boldsymbol{K_h} + \sigma_v^2 \boldsymbol{K_v} \\ \boldsymbol{K_h} + \frac{\sigma_v^2}{M\sigma_p^2} \boldsymbol{K_v} & \sigma_d \boldsymbol{K_h} \end{array} \right], \quad (18)$$

which can be rewritten as

$$\boldsymbol{AK_r} = \frac{1}{2} \left(\left[\begin{array}{cc} \sigma_d & 0 \\ 0 & 1 \end{array} \right] \otimes \boldsymbol{I_L} \right) \left[\begin{array}{cc} \boldsymbol{K_h} & \boldsymbol{K_h} + \frac{\sigma_v^2}{\sigma_d^2} \boldsymbol{K_v} \\ \boldsymbol{K_h} + \frac{\sigma_v^2}{M\sigma_p^2} \boldsymbol{K_v} & \boldsymbol{K_h} \end{array} \right]$$
$$\times \left(\left[\begin{array}{cc} 1 & 0 \\ 0 & \sigma_d \end{array} \right] \otimes \boldsymbol{I_L} \right). \quad (19)$$

Now, noting that the eigenvalue decomposition is commutative with respect to matrix multiplication and that it is

$$\left(\left[\begin{array}{cc} 1 & 0 \\ 0 & \sigma_d \end{array} \right] \otimes \boldsymbol{I_L} \right) \left(\left[\begin{array}{cc} \sigma_d & 0 \\ 0 & 1 \end{array} \right] \otimes \boldsymbol{I_L} \right) = \sigma_d \boldsymbol{I_{2L}} \quad (20)$$

and, finally, that scaling all eigenvalues by the same factor does not effect the bit error probability, as can bee seen in (13), it can be stated that the eigenvalues λ_l in (13) and (16) can be determined by the eigenvalue decomposition of

$$\left[\begin{array}{cc} \boldsymbol{K_h} & \boldsymbol{K_h} + \frac{1}{\rho_d} \boldsymbol{K_v} \\ \boldsymbol{K_h} + \frac{1}{\gamma\rho_d} \boldsymbol{K_v} & \boldsymbol{K_h} \end{array} \right]. \quad (21)$$

Note that the matrix in (21) depends on the key system parameters

$$\rho_d = \frac{\sigma_d^2}{\sigma_v^2} , \quad \gamma = M \frac{\sigma_p^2}{\sigma_d^2}, \quad (22)$$

denoting the data bit-energy-to-interference ratio and the effective pilot-energy-to-data-energy ratio, respectively, the channel covariance matrix $\boldsymbol{K_h}$ and the interference covariance matrix $\boldsymbol{K_v}$.

IV. APPLICATION TO W-CDMA

In this Section we consider the application of the general results from the previous section to the uplink of a W-CDMA system [16] employing multiple antenna elements at the basestation and examine the effect of spatial colored interference. The receiver structure is illustrated in Figure 1. A simple fixed beamforming technique is applied [13]. A limited number of Q static beams are steered in different directions to cover a $120°$ sector as it is illustrated in Figure 2 for a 4 element uniform linear array (ULA) with $\lambda/2$ inter-element spacing. The number of beams is assumed to equal the number of antenna elements and the beam directions are assumed to be equi-spaced resulting in the set of steering directions $\{-45°, -15°, 15°, 45°\}$.

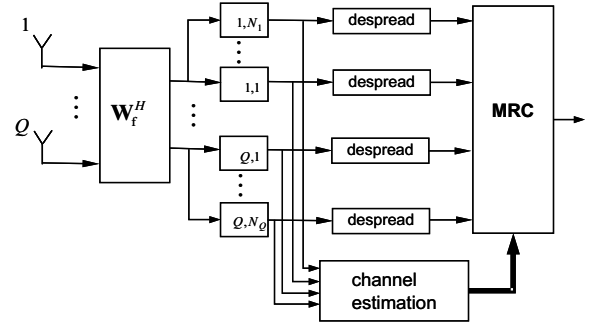The beamforming operation can be expressed in terms of a

Fig. 1. Fixed Beamformer Receiver Structure.

$(Q \times Q)$ linear transformation of the antenna outputs, where the columns of $\boldsymbol{W}_f$ are the respective fixed beamforming vectors. For the relevant channel tap delays the best beam outputs are selected and used for maximum ratio combining.

A. W-CDMA Spatio-Temporal Signal Model

The following spatio-temporal signal model is applied. In the delay domain, the channel of each of the K users is modelled as a tapped delay line with L_k, $k = 1, \ldots, K$, independent Rayleigh fading channel taps and normalized total channel power, i.e. $\sum_{l=1}^{L_k} \sigma_{l,k}^2 = 1$, where $\sigma_{l,k}^2$ is the mean channel tap power of the l-th channel tap of user k. Since the focus is on the effects of the spatial correlations and for the ease of notation, the multipath delays $\tau_{l,k}$ are assumed to be integer multiples of the chip period T_c, i.e. $\tau_{l,k} = d_{l,k} T_c$ with $d_{l,k}$ integer.

Hence, the T_c-sampled version of the received signal after pulse-matched filtering at the $q-th$ antenna element can be written as

$$z_n^{(q)} = z^{(q)}(nT_c) = \sum_{k=1}^{K} \sum_{l=1}^{L_k} h_{l,k}^{(q)} b_{n-d_{l,k},k} + v_n^{(q)}, \quad (23)$$

where n is the time index and $v_n^{(q)}$ is AWGN with variance σ_v^2. The quantity $b_{n-d_{l,k},k}$ denotes the k-th user's transmitted QPSK modulated sequence [15] delayed by $d_{l,k}$ chips consisting of the scrambled superposition of the binary pilot signal [dedicated physical control channel (DPCCH)] and the data signal [dedicated physical data channel (DPDCH)]

$$b_{n,k} = c_{\text{scram},n,k}(c_{\text{data},n,k} c_k + j\beta c_{\text{pilot},n,k})/\sqrt{1 + \beta^2}, \quad (24)$$

where β is the pilot-to-data amplitude ratio, $c_{\text{scram},n,k}$ denotes the complex scrambling sequence, $c_{\text{data},n,k}$ and $c_{\text{pilot},n,k}$ denote the binary data and pilot spreading sequences, and c_k is the k-th user's transmitted data symbol. Note that it is $|b_{n,k}|^2 = 1$. Due to the Rayleigh fading assumption, the channel coefficients are completely characterized by their second order statistics

$$\mathrm{E}\{h_{l,k}^{(q)} h_{l',k'}^{(q')*}\} = \sigma_{l,k}^2 \boldsymbol{K}_{\text{S},l,k}(q, q') \delta(l, l') \delta(k, k'), \quad (25)$$

where $\delta(\cdot)$ denotes the Kronecker delta and $\boldsymbol{K}_{\text{S},l,k}(q, q')$ denotes the element of the spatial correlation matrix $\boldsymbol{K}_{\text{S},l,k}$

corresponding to the q-th row and q'-th column and the l-th channel path of user k

$$\boldsymbol{K}_{S,l,k} = \int \boldsymbol{a}(\theta)\boldsymbol{a}(\theta)^H f_{\theta_{l,k}}(\theta)\, d\theta. \tag{26}$$

The quantity $f_{\theta_{l,k}}(\theta)$ denotes the angular power density function accounting for an angular spreading of the signal energy. For a uniform linear array with inter element spacing of $\Delta = 1/2$ wavelengths, the array response vector is given by

$$\boldsymbol{a}(\theta) = [1, e^{-j2\pi\Delta\sin\theta}, \dots, e^{-j2\pi(Q-1)\Delta\sin\theta}]. \tag{27}$$

Note that the diagonal elements of $\boldsymbol{K}_{S,l,k}$ are unity.

B. Bit Error Probability

In order to derive an expression of the signal vector prior to maximum ratio combining in Figure 1 we note that the beamforming and despreading operations can be interchanged. Let the desired user correspond to $k = 1$, then the output after despreading at the q-th antenna element corresponding to the l-th channel tap is given by

$$
\begin{aligned}
y_l^{(q)} &= \frac{1}{\sqrt{N_d}} \sum_{n=1}^{N_d} c_{\text{data},n-d_{l,1},1} c_{\text{scram},n-d_{l,1},1}^* z_n^{(q)} \\
&= \sqrt{\frac{N_d}{1+\beta^2}} h_{l,1}^{(q)} c_1 \\
&+ \frac{1}{\sqrt{N_d}} \sum_{\substack{l'=1 \\ l'\neq l}}^{L_1} h_{l',1}^{(q)} \sum_{n=1}^{N_d} b_{n-d_{l',1},1} c_{n-d_{l,1},1}^* \\
&+ \frac{1}{\sqrt{N_d}} \sum_{k=2}^{K} \sum_{l=1}^{L_k} h_{l,k}^{(q)} \sum_{n=1}^{N_d} b_{n-d_{l,k},k} c_{n-d_{l,1},1}^* \\
&+ \frac{1}{\sqrt{N_d}} \sum_{n=1}^{N_d} v_n^{(q)} c_{n-d_{l,1},1}^*,
\end{aligned} \tag{28}
$$

where N_d is the data spreading factor and the $c_{n-d_{l,1},1} = c_{\text{data},n-d_{l,1},1} c_{\text{scram},n-d_{l,1},1}$ denote the effective chips used for despreading. Note that the first term in (28) is the desired component, the second term is the interpath interference (IPI) component, the third term characterizes the multiple access interference (MAI) and the last term is AWGN.

Since the scrambling code in the uplink is a fraction of a very long Gold sequence (length $\approx 2^{24}$) [15], the imperfect autocorrelation properties of the pseudo noise sequences can be statistically described by making use of the following common approximation [14]:

$$h_{l',k}^{(q)} \frac{1}{\sqrt{N_d}} \sum_{n=1}^{N_d} b_{n-d_{l',k},k} c_{n-d_{l,1},1}^* \tag{29}$$

is approximately $\mathcal{N}_C(0, \sigma_{l',k}^2)$ distributed for $k > 1$ or $l' \neq l$. By defining $\boldsymbol{y}_l = [y_l^{(1)}, \dots, y_l^{(Q)}]^T$, the signal vector of the desired user corresponding to the l-th channel tap after despreading is given by

$$\boldsymbol{y}_l = c_1 \sqrt{\frac{N_d}{1+\beta^2}} \widetilde{\boldsymbol{h}}_l + \widetilde{\boldsymbol{v}}_l, \tag{30}$$

with $\widetilde{\boldsymbol{h}}_l$ the desired component and $\widetilde{\boldsymbol{v}}_l$ the interference component in (28) being zero-mean complex Gaussian distributed. The respective covariance matrices are given by

$$\boldsymbol{K}_{\widetilde{\boldsymbol{h}}_l} = \boldsymbol{K}_{S,l,1} \tag{31}$$

$$\boldsymbol{K}_{\widetilde{\boldsymbol{v}}_l} = \sum_{\substack{l'=1 \\ l'\neq l}}^{L_1} \sigma_{l',1}^2 \boldsymbol{K}_{S,l',1} + \sum_{k=2}^{K} \sum_{l=1}^{L_k} \sigma_{l,k}^2 \boldsymbol{K}_{S,l,k} + \sigma_v^2 \boldsymbol{I}_Q. \tag{32}$$

Now, accounting also for the fixed beamforming operation we define

$$
\begin{aligned}
\boldsymbol{x}_l &= \boldsymbol{J}_l^T \boldsymbol{W}_f^H \boldsymbol{y}_l \\
&= c_1 \sqrt{\frac{N_d}{1+\beta^2}} \boldsymbol{J}_l^T \boldsymbol{W}_f^H \widetilde{\boldsymbol{h}}_l + \boldsymbol{J}_l^T \boldsymbol{W}_f^H \widetilde{\boldsymbol{v}}_l \\
&= c_1 \sqrt{\frac{N_d}{1+\beta^2}} \breve{\boldsymbol{h}}_l + \breve{\boldsymbol{v}}_l,
\end{aligned} \tag{33}
$$

where $\boldsymbol{J}_l$ is a selection matrix with only one element being "1" in each row while the other elements are all "0". The matrix $\boldsymbol{J}_l$ accounts for the fact that except from a spatially uncorrelated scenario only a subset of beams is used for maximum ratio combining with respect to the l-th channel tap. These are usually the beams steering into the direction of arrival of the channel taps.
Due to the linear transformation with $\boldsymbol{J}_l^T \boldsymbol{W}_f^H$, $\breve{\boldsymbol{h}}_l$ and $\breve{\boldsymbol{v}}_l$ are also zero-mean complex Gaussian distributed and characterized by the respective covariance matrices.

$$\boldsymbol{K}_{\breve{\boldsymbol{h}}_l} = \boldsymbol{J}_l^T \boldsymbol{W}_f^H \boldsymbol{K}_{S,l,1} \boldsymbol{W}_f \boldsymbol{J}_l \tag{34}$$

$$\boldsymbol{K}_{\breve{\boldsymbol{v}}_l} = \boldsymbol{J}_l^T \boldsymbol{W}_f^H \boldsymbol{K}_{\widetilde{\boldsymbol{v}}_l} \boldsymbol{W}_f \boldsymbol{J}_l. \tag{35}$$

The total input diversity vector used for maximum ratio combining is given by stacking the vectors $\boldsymbol{x}_1, \dots, \boldsymbol{x}_{L_1}$ of all channel taps on top of each other

$$
\begin{aligned}
\boldsymbol{x} &= \left[\boldsymbol{x}_1^T, \dots, \boldsymbol{x}_{L_1}^T \right]^T \\
&= c_1 \sqrt{\frac{N_d}{1+\beta^2}} \breve{\boldsymbol{h}} + \breve{\boldsymbol{v}},
\end{aligned} \tag{36}
$$

with $\breve{\boldsymbol{h}} = [\breve{\boldsymbol{h}}_1^T, \dots, \breve{\boldsymbol{h}}_{L_1}^T]^T$ and $\breve{\boldsymbol{v}} = [\breve{\boldsymbol{v}}_1^T, \dots, \breve{\boldsymbol{v}}_{L_1}^T]^T$, respectively. Note that the covariance matrices $\boldsymbol{K}_{\breve{\boldsymbol{h}}}$ and $\boldsymbol{K}_{\breve{\boldsymbol{v}}}$ of $\breve{\boldsymbol{h}}$ and $\breve{\boldsymbol{v}}$ are block-diagonal, since the channel taps have been assumed to be mutually uncorrelated.
The derivation for the ML-channel estimate is analogous. Finally, if a block of N_p pilot chips is used for the channel estimation, then the bit error probability is given by (16) with the λ_l being the eigenvalues of

$$
\begin{bmatrix}
\boldsymbol{K}_{\breve{\boldsymbol{h}}} & \boldsymbol{K}_{\breve{\boldsymbol{h}}} + \frac{1+\beta^2}{N_d} \boldsymbol{K}_{\breve{\boldsymbol{v}}} \\
\boldsymbol{K}_{\breve{\boldsymbol{h}}} + \frac{1+\beta^2}{\beta^2 N_p} \boldsymbol{K}_{\breve{\boldsymbol{v}}} & \boldsymbol{K}_{\breve{\boldsymbol{h}}}
\end{bmatrix}, \tag{37}
$$

where (37) is in accordance with (21).

C. Example

In the following a 2 user scenario is considered in order to demonstrate the effect of colored interference. The antenna configuration and the choice of the fixed beam matrix $\boldsymbol{W}_f$ is as illustrated in Figure 2. A 2 tap fading channel with channel tap powers $\{0, -3\}$ dB, channel tap delays $\{0, 1\}T_c$, direction of arrival $\{15°, 15°\}$ and an uniform angular spread of $5°$
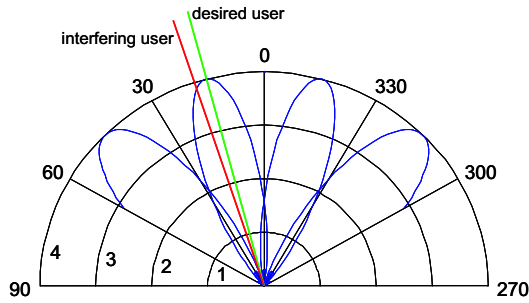
Fig. 2. Considered 2 user scenario: desired user at $15°$, interfering user at $18°$. Antenna configuration: $Q = 4$ antenna elements with beam steering directions $\{-45°, -15°, 15°, 45°\}$.

is considered for the desired user. According to the 3GPP standard [16] the data spreading factor is set to $N_d = 64$ and the pilot-to-data ratio is set to $\beta = 11/15$, according to a typical speech user. The ML-channel estimation is performed by coherently accumulating over 6 pilot symbols, i.e. $N_p = 1536$ chips. A flat fading channel has been assumed for the interfering user.

By evaluating (16), the mean bit error probability is plotted versus the bit-energy-to noise ratio ($E_b/N_0 = N_d/(1 + \beta^2)/\sigma_v^2$) in Figure 3. It is assumed that only the fixed beam pointing into the direction of the desired user is used for maximum ratio combining. The interferer is assumed to have the same E_b/N_0 as the desired user.

The dotted curve shows the result if no IPI and no MAI are considered. The dashed curve also considers only the desired user but accounts for IPI. As expected, for high E_b/N_0 a suturation effect can be observed. If the fading coefficients of the interferer are spatially fully correlated at the antenna elements (the ○ markers), the saturation effect worsens significantly. On the other hand, if the interferer fading coefficients are spatially uncorrelated (the × markers) only a slight degradation can be observed.

This is due to the fact that the fixed beam performs some kind of interference suppression by rejecting the interference power coming from directions other than that of the desired user. However, in case of a spatially correlated interferer a rejection of the interference power is not possible, since the DOA of the interferer is too close to the DOA of the desired user.

However, even in the case of a spatially correlated interferer, a performance gain of nearly 6 dB is obtained compared to the one antenna case. This is due to the fact that the DOA of the desired user matches the steering direction of a fixed beam and hence, nearly the full antenna array gain can be exploited.

V. Conclusion

An analytical closed form expression for the mean bit error probability of a binary modulated signal has been presented for the general case of maximum ratio combining with imperfect channel state information accounting also for colored additive interference. Furthermore, an alternative solution has been derived, which allows a numerically stable evaluation of the bit error probability also for the case of closely-spaced eigenvalues of the respective channel and interference covariance
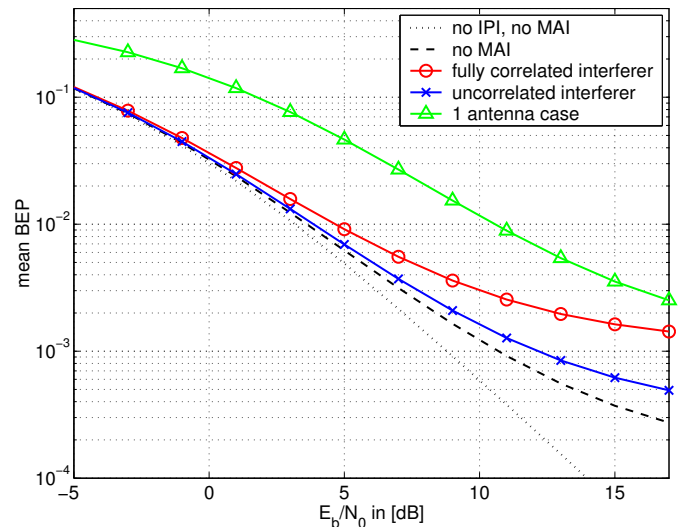

Fig. 3. BER versus E_b/N_0 for a 2-tap multipath fading channel and one flat fading interfering user.

matrices.

The effect of colored interference has been demonstrated by applying the results to a multi antenna element receiver in the uplink of a W-CDMA system, where the signal reception is disturbed by interpath and multiple access interference.

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