Evaluation of Beamforming for Broadcast Applications in Single Frequency Networks

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***Abstract*— In this paper, the application of beamforming in a single frequency network (SFN) is examined. A single frequency network is characterized by the transmission of the same signal from multiple base stations simultaneously. Since a user receives the signal from many base stations, the beamforming weights for many different base stations need to be jointly optimized for the transmission. An optimality criterion is given and three different beamforming strategies with varying computational complexity and performance are investigated. Their performances are compared to the case of SFN transmission without beamforming, showing significant gains for realistic numbers of transmit antennas at the base stations and numbers of users in the SFN.**

I. Introduction

The transmission of the same content to a multitude of users is called broadcast herein and is opposed to the transmission to a single user, which is called unicast. Typical applications of such a scenario include the streaming of video or audio content to users. Broadcast has been considered under the name MBMS (Multimedia Broadcast/Multicast Service) (cf. [1]) as a feature of UMTS and under the name MBS (Multicast and Broadcast Service) as a feature of WiMAX (cf. [2]) to provide the network operators with an additional service. The existence of an antenna array at the base station in such systems is not uncommon, and, as such, has not only been considered for the application to unicast transmission but also to increase the performance of broadcast transmission. Most often, the antenna elements at the base station will be designed to have a low correlation, as e.g. having a large normalized distance or being cross-polarized. In these cases, the antennas can be used to provide a gain [3][4][5], where the antenna weights are adapted with a rate depending on the fast fading of the users, which requires fast feedback from all users participating in the broadcast reception. If the antenna elements at the base station have a low separation such as half a wavelength, the antenna elements may be used for transmit beamforming. This requires only the knowledge of long-term signal properties such as the spatial correlation matrix and is treated in this paper.

The focus of this paper lies in the applicability of the beamforming technique to upcoming mobile radio standards like evolved UTRAN or eUTRAN, which will feature OFDM as the employed downlink transmission technique. OFDM allows for the transmission of broadcast data in a so-called single frequency network, in which all participating base stations transmit the same signal simultaneously. If the cyclic prefix of the OFDM symbols is long enough, the components from different base stations will be constructively combined during reception, facilitating the exploitation of macro-diversity without further signal processing. While the state-of-the-art in the literature has predominantly considered beamforming for single cell transmission, where signal reception is worst at the cell borders, this paper evaluates the use of beamforming in an SFN setting, where no cell borders in the original sense exist. In order to apply beamforming in an SFN setting, an optimality criterion is defined and a solution according to this criterion is considered as a first beamforming strategy. Since finding a solution fulfilling this optimality criterion is computationally very complex and requires communication between the involved base stations, two additional suboptimal strategies are also considered. These suboptimal strategies have a lower computational complexity at the expense of a relatively slight performance decrease. These algorithms seem manageable regarding the computational burden and required network infrastructure.

II. System Setup

A part of an SFN is regarded here. This part consists of N_C cells, which are being served by beamforming capable base stations (BSs). The rest of the SFN is assumed to consist of cells that are being served by omnidirectionally transmitting base stations. The beamforming capable base stations have N_T antennas each per cell and N_U users, which are being referred to as user equipments (UEs) in the context of eUTRAN, are distributed uniformly in the area consisting of these N_C cells. Only users in these N_C cells are being regarded here (cf. Fig. 1).

The transmission is assumed to be noise limited, which e.g. corresponds to the case of OFDM transmission with a sufficient cyclic prefix length and synchronization (cf. [7]). In this case, the contributions from different base stations arriving at the user may be regarded as multipath contributions from a single base station. Because of the assumption of a cyclic

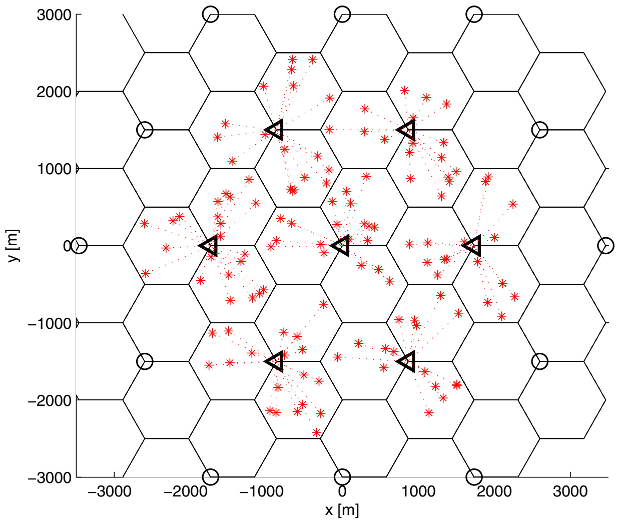

Fig. 1. Grid with base stations in a three-cell layout. Base stations with antenna arrays are depicted by a triangle, omnidirectionally transmitting base stations by a circle, users by a star, $N_C = 21$, $N_U = 128$

prefix longer than the effective power delay profile (PDP), the average SNR of a particular user i may then be represented as the sum over all these contributions:

$$\gamma_i = \sum_{c=1}^{N_C} \mathbf{w}_c^H R_{sp,i}^c \mathbf{w}_c + \gamma_{omni,i}, \tag{1}$$

where $\mathbf{w}_c$ is the beamforming vector of cell c, $\gamma_{omni,i}$ is the SNR provided by the omnidirectionally transmitting base stations and the noise variance is set to unity without loss of generality.

We assume that channel coefficients from different base stations are uncorrelated, i.e.

$$\mathrm{E}\{h_{i,m}^c \cdot h_{i,n}^{d*}\} = 0, \quad c \neq d \tag{2}$$

which should be easily fulfilled because of the geographical separation of the base stations from each other. The channel coefficient $h_{i,m}^c$ denotes here the channel to user i from antenna element m of base station c. Channel coefficients from antennas of a single base station antenna array however are assumed to be correlated according to

$$R_{sp,i}^c = \mathrm{E}\{\mathbf{h}_i^c \cdot \mathbf{h}_i^{cH}\} \tag{3}$$

with $R_{sp,i}^c$ denoting the spatial correlation matrix of user i as seen from cell c. The channel vector $\mathbf{h}_i^c$ between user i and the antenna array with N_T elements at base station c is given as

$$\mathbf{h}_i^c = [h_{i,1}^c, \dots, h_{i,N_T}^c]^T \tag{4}$$

The restriction to N_C beamforming capable cells is mainly due to simulation complexity reasons but can also represent the case that only a patch of the whole network is actually in possession of beamforming capable base stations.

As the considered transmission scheme is OFDM and as such a multicarrier transmission technique, the long-term SNR will in general be a function of the subcarrier index because of a possible frequency dependency of the spatial correlation matrix. In this paper, the spatial correlation matrix is assumed to be independent on the frequency. A frequency dependency of the spatial correlation matrix can be observed if the bandwidth of the system is not small compared to the carrier frequency as in ultra-wideband (UWB) systems. In the case that the frequency dependency is not negligible, a straightforward generalization of the beamforming algorithms can be obtained by extending the optimization to all subcarriers.

III. Algorithms

In this section, three different algorithms are presented that vary in their complexity and performance.

A. Network-wide Optimization

As the broadcast should be received by as many users as possible and thus, the data rate has to be chosen based on the poorest link, the beamforming weights should be chosen such that the minimum occurring SNR is maximized:

$$\mathbf{W}_{opt} = \arg\max_{\mathbf{W}} \min_i \left(\sum_{c=1}^{N_C} \mathbf{w}_c^H R_{sp,i}^c \mathbf{w}_c + \gamma_{omni,i}\right), \tag{5}$$

where the maximization is done over the matrix of beamforming weights $\mathbf{W}$ with

$$\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_{N_C}], \quad \mathbf{w}_c^H \mathbf{w}_c = 1 \ \forall\ c \tag{6}$$

The maximization of the minimum SNR can be performed regarding all users in the N_C cells. However, if a single user happens to be in poor reception conditions, the overall system performance will decrease since many resources are used to improve this poor link. Therefore, it is important to consider a more robust optimization algorithm. Instead of maximizing the minimum SNR of every user in the regarded area, only the part P_{user} having the highest SNR can be used in the optimization process. This excludes the worst $1 - P_{user}$ from the optimization, yielding a better performance for the remaining part. This part P_{user} can be chosen in accordance to coverage requirements from standards specifications.

Problems of the above-mentioned type belong to the group of nonlinear programs and can be solved iteratively by sequential quadratic programming (SQP) [6]. The basic principle of SQP is that is solves the original problem via the minimization of a series of quadratic subproblems.

Sequential quadratic programming can not guarantee to converge to the global optimum, instead, it may converge to a local one. With the initialization of

$$\mathbf{w}_c = [1, 0, \dots, 0] \ \forall\ c, \tag{7}$$

which resembles omnidirectional transmission, a performance increase compared to omnidirectional transmission can be achieved after the application of SQP, even if the global optimum could not be reached.

B. Isolated Optimization

Finding the solution of Eq. (5) requires knowledge of every user's correlation matrix and every cell's beamforming vector in the entity that performs the optimization. This necessitates a transmission of these quantities to the optimizing entity and a transmission of the resulting beamforming weights back to the respective base stations. Furthermore, the computational complexity is very high, so that this beamforming strategy, even though it works only on long-term signal properties, seems to be difficult to implement in practice.

Therefore, two low-complexity beamforming strategies are considered additionally. The first strategy is the isolated case, where a beamforming weight is determined by every base station irrespective of other base stations contributions:

$$\mathbf{w}_{iso,c} = \arg\max_{\mathbf{w}_c} \min_i \mathbf{w}_c^H R_{sp,i}^c \mathbf{w}_c \quad \forall\ c, \tag{8}$$

and the minimization is only made for the users in the regarded cell c. This means that the optimization is completely unaware of the contributions of other base stations to the SNR of each user.

Just as with the network-wide optimization algorithm, it is possible to include all users in the regarded cell in the optimization or only the part P_{user} with the strongest links.

C. Recursive Optimization

While the above-mentioned strategy does not take contributions to the SNR from other base stations into account, the strategy described in this subsection does take these into account. Here, the total SNR observed by every user is reported to its associated base station, which calculates the extra SNR $\gamma_{x,i}$ received by this user from other base stations with the knowledge of the spatial correlation of the user and the base station's beamforming weights. Then, the total SNR of the users in cell c is optimized by a proper choice of the base station beamforming weights.

$$\mathbf{w}_{net,c} = \arg\max_{\mathbf{w}_c} \min_i (\mathbf{w}_c^H R_{sp,i}^c \mathbf{w}_c + \gamma_{x,i}) \quad \forall\ c \tag{9}$$

Eq. (9) requires the knowledge of the extra SNR $\gamma_{x,i}$ for the determination of the beamforming weights, which in turn itself depends on the beamforming weights of the other base stations. Therefore, a recursive algorithm is used for the determination of the weights to converge to a stable operating point:

$$\mathbf{w}_{rec,c}(k+1) = \mathbf{w}_{rec,c}(k) + \mu \cdot [\mathbf{w}_{net,c}(k) - \mathbf{w}_{rec,c}(k)] \tag{10}$$

For a good trade-off between convergence speed and accuracy, a step-size parameter μ between 0.3 and 0.5 showed to be a good choice, resulting in a number of required iterations of about five to ten for convergence.

Here it is also possible to include all users in the regarded cell in the optimization or only the strongest P_{user} ones.

IV. RESULTS

Table I shows the major system parameters that are being used unless otherwise noted.

Parameter	**Value**
Carrier frequency	2 GHz
Intersite distance	1732 m
Sectorized	Yes
Sectorization pattern	-10 dB at 60°
Path loss exponent	3.76
Number of UEs	128
Number of beamforming BSs	21
User placement	uniform (cartesian)
Transmit antennas	4
Antenna spacing	half-wavelength
Power angular density	Laplacian, 5° std. dev.

TABLE I
DEFAULT SIMULATION PARAMETERS

While in standard unicast traffic, the cell edge regions are the areas with the poorest reception conditions, in SFN operation the whole area can be understood as a single cell and the distribution of the SNR across the plane is not as intuitive.

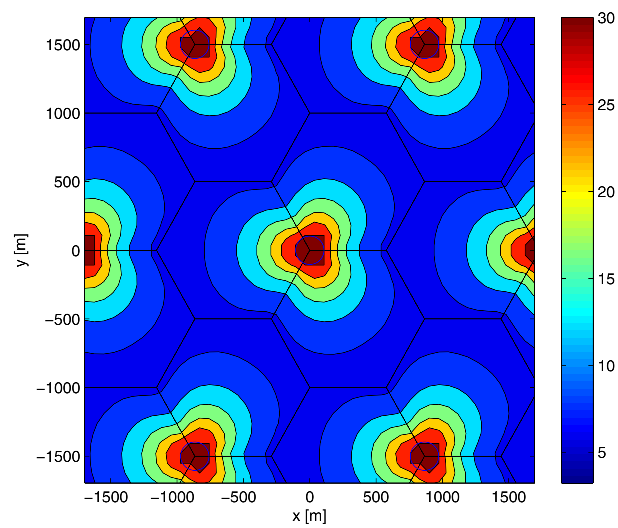

Fig. 2. Average total SNR [dB] as a function of the location, omnidirectional transmission

Figure 2 shows the long-term SNR for omnidirectional transmission in the case of SFN operation, including both the contributions of the own cell as well as contributions from all other cells. It shows a comparable distribution as in the unicast non-SFN case in that areas near to cell borders yield the lowest SNR.

In these areas, contributions from other cells become significant as is shown in Figure 3. Since the areas of lowest SNR are located near to the cell border, where contributions from more than one base station are significant, a determination of the beamforming weights in a joint fashion for all base stations simultaneously can be expected to yield better results than a determination for each base station separately.

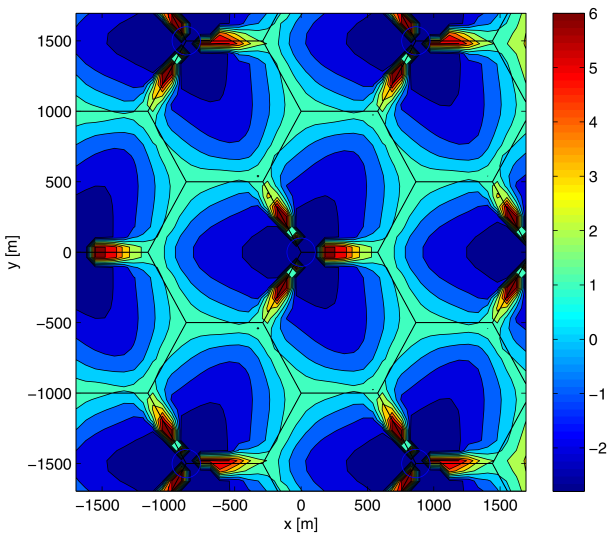

Fig. 3. Contributions to the total SNR [dB] from adjacent BSs as a function of the location, omnidirectional transmission

In order to evaluate the performance of the proposed strategies, the complementary cumulative density function (ccdf) of the users' SNR is given in Fig. 4. The ccdf is shown for high percentages of users as broadcast is typically required to be received by more than 90% of the users in a system. The figure shows an improvement in the order of 1.7 dB for all strategies with this constellation of $N_T = 4$ transmit antennas and $N_U/N_C = 128/21 \approx 6$ users per cell compared to omnidirectional transmission. The network-wide optimization outperforms the two suboptimal strategies especially for percentages near 100%, but does so at the expense of performance at lower percentages.

If not all users are taken into account in the calculation of the beamforming weights but only $P_{user} = 0.9$, the network-wide optimization outperforms the other strategies for these 90% of the users by about 0.1 dB and 0.4 dB as shown in Figure 5. Furthermore, the SNR for 90% of the users with this relaxation is about 0.5 dB higher compared to the case where all users are taken into account for the determination of the beamforming weights ($P_{user} = 1$, cf. Figure 4).

Obviously, the benefit of an antenna array for the transmission to a number of users depends on the number of antenna elements per base station and on the number of users in the area. If the number of users per cell is significantly larger than

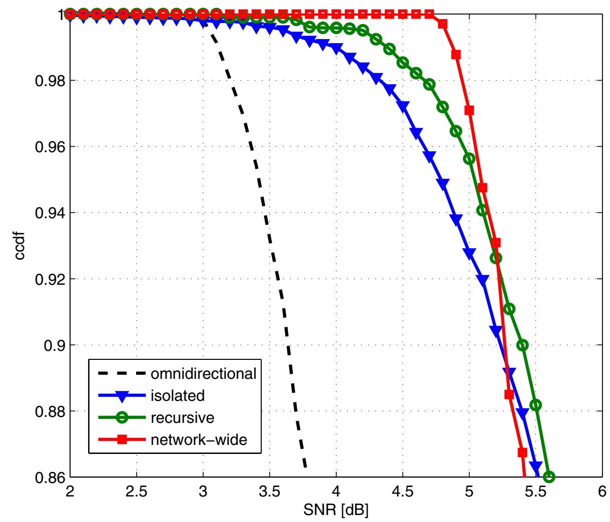

Fig. 4. Complementary cumulative distribution function of the SNR of the users within the N_C cells, $N_U = 128$, $N_T = 4$, $N_C = 21$, $P_{user} = 1$

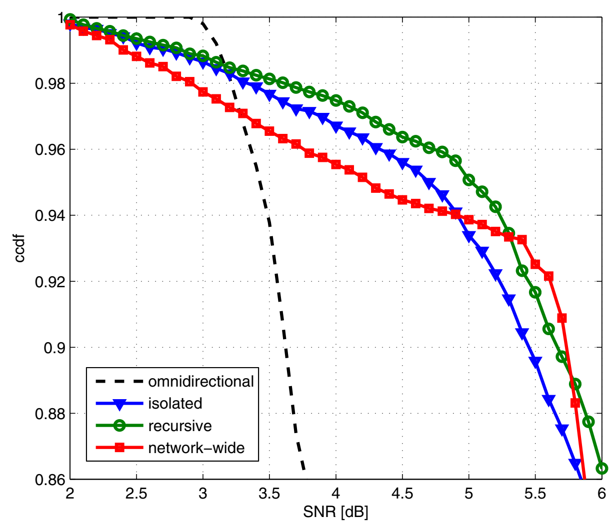

Fig. 5. Complementary cumulative distribution function of the SNR of the users within the N_C cells, $N_U = 128$, $N_T = 4$, $N_C = 21$, $P_{user} = 0.9$

the number of antenna elements, no gain can be expected since this would necessitate an omnidirectional transmission. On the other hand, if the number of users is very low, e.g. one, all available power could be radiated towards this user, yielding a high gain for this user. Figure 6 shows the minimum SNR for the best 90% of the users in the area, where the number N_U is varied from 32 to 256 and the number of transmit antennas N_T is held constant at 4. It can be seen that a difference between the three proposed beamforming strategies exists primarily for lower numbers of users. However, even with a high number of users, beamforming yields a gain compared to omnidirectional transmission. For $N_U = 256$ corresponding to about twelve users per cell, a gain of about 1.5 dB can be achieved with the employed four-element arrays.

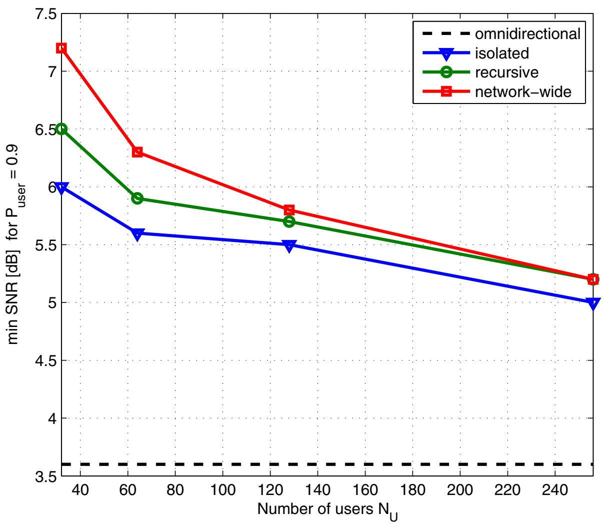

Fig. 6. Minimum SNR [dB] for $P_{user} = 0.9$ of all N_U users in the area

V. Conclusion and Outlook

Beamforming has been shown to be an effective way to increase the SNR of the worst links in a broadcast scenario within a single frequency network. A high complexity strategy and two suboptimal strategies with a lower complexity have been proposed and their performances shown. Even the simplest strategy yields a large performance benefit compared to omnidirectional transmission. The improved link quality reduces the block error rate or the number of retransmissions if ARQ is being used. Alternatively, the amount of transmit power for the broadcast transmission can be lowered so as to provide more power for a unicast transmission, if unicast and broadcast is transmitted from the base stations simultaneously. Even though the benefit depends on the number of users and the number of antenna elements, significant gains can even be achieved for dense user populations and a low number of antenna elements. The derivation and evaluation of algorithms with a lower computational complexity will be pursued in future work.

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