Downlink based Intercell Time Synchronization using Maximum Likelihood Estimation

Markus Jordan, Martin Senst, Yang Cui, Gerd Ascheid and Heinrich Meyr
Institute for Integrated Signal Processing Systems
RWTH Aachen University
phone-number: +49 (241) 80 27875
e-mail: jordan@iss.rwth-aachen.de

***Abstract*— A method to establish a time synchronization for base stations in a cellular network is derived based on the maximum likelihood estimation principle. Its performance is analyzed and compared to a heuristic technique from the literature. The main feature of these synchronization algorithms is that a time synchronization of the base stations is achieved by measurements from the users in the downlinks that are associated to the base stations. The steady-state is reached after a low number of iterations and a low residual error is obtained. The good accuracy allows the application of this algorithm in the context of single frequency networks (SFN), where a high degree of time-synchronization between participating base stations is required.**

I. Introduction

In order to provide broadcast data to an area larger than one cell, single frequency networks have been shown to yield large performance benefits compared to single cell transmission of broadcast data from many unsynchronized cells. The preferred transmission scheme for single frequency networks is OFDM, since this technique copes well with the resulting highly time dispersive channel, where useful signal components arrive at the user from many base stations which are potentially far away. In addition to the delay spread of the channel that is caused by the propagation time, imperfect time synchronization of the base stations affects the delay spread too, resulting in an even larger spread. The afore-mentioned large gains of SFN transmission compared to single cell transmission can however only be achieved if the channel delay spread is not significantly larger than the cyclic prefix (CP) length of the OFDM symbols. Otherwise, if signal components from different base stations arrive at a UE with a temporal separation larger than the CP length, a part of the total signal power will act as interference in the reception and thus degrade the overall link quality.

The focus of this paper lies in the applicability of the proposed algorithm to the upcoming standard for wireless mobile networks called evolved UTRAN or eUTRAN, which will feature OFDM as the downlink transmission technique. Evolved UTRAN will feature the broadcast of data in a service entitled multimedia broadcast/multicast service (MBMS) and will include the possibility of a transmission of this service in an SFN.

Various ways have been proposed to enable a temporal synchronization of base stations in the order of the CP length, i.e. with an accuracy of about one microsecond in the case of eUTRAN. The most often mentioned technique is a time synchronization with GPS equipment in the base stations. Time synchronization via GPS is very accurate, but the reliance on third parties that provide a GPS signal represents a disadvantage of this technique. Furthermore, GPS reception is usually only possible with line of sight to the transmitting satellites, which prohibits GPS reception for indoor base stations or base stations surrounded by tall obstacles.

In contrast to this technique, a heuristic synchronization technique relying solely on the wireless communication system that is to be synchronized has been proposed in [2]. The main idea of this scheme is that participating users can measure a time difference between their own base station and the surrounding base stations and transmit this measurement to their own base station. Every base station can then adapt its time base according to these measurements. Furthermore, the proposed synchronization algorithm is also suitable for self-organizing networks or ad-hoc networks, where time synchronization of access nodes can be used for interference avoidance. In this paper, the same system setup is assumed but the synchronization algorithm is deduced from the maximum likelihood principle. The herein proposed algorithm performs significantly better than the technique described in [2] and requires only little further information.

This paper is structured as follows: After this introduction, the system model is shown in Section II. The maximum likelihood estimator is derived in Section III. Section IV evaluates for which kinds of distributions the presented algorithm actually represents the maximum likelihood estimator. Section V shows the simulation results for both algorithms and Section VI concludes this paper.

II. System Setup

The regarded system as shown in Fig. 1 consists of $N_C = 19$ hexagonal one-sector-site cells of side length $a = 1000\,\mathrm{m}$, resulting in an intersite distance of $ISD = 1732\,\mathrm{m}$. A total of N_U users is distributed uniformly in Cartesian coordinates within the whole area. In this paper, only direct path propagation of the radio waves is considered and the position of the mobiles is assumed to be fixed throughout the simulation duration as in [2]. The base stations transmit pilot symbols to enable a time synchronization between the users of the cell and the associated base station.

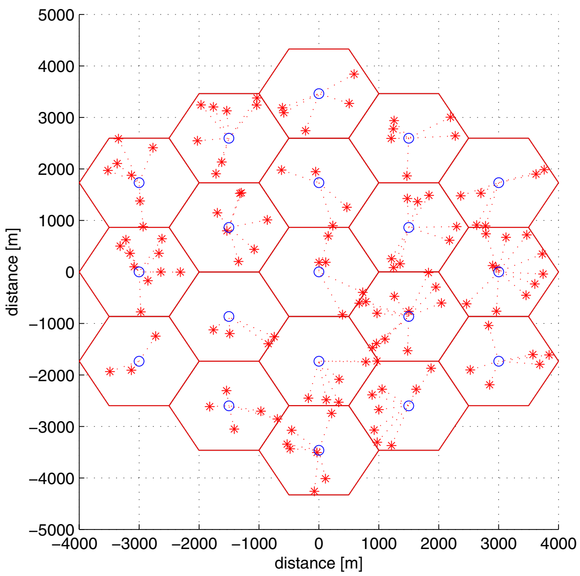

Fig. 1. Grid with omnidirectional base stations, $N_C = 19$, $N_U = 128$, $ISD = 1732\,\mathrm{m}$

In addition to synchronizing to their own base station, i.e. to the base station whose transmission is received with the highest power, the users also estimate the time offset between their own base station and the surrounding base stations using these pilot symbols. This estimation is assumed to be perfect with the exception of the influence of the propagation time, so that the effect of this systematic error is emphasized. Users feed back this estimated offset to their own base station to enable the intercell time synchronization which is described in the next section.

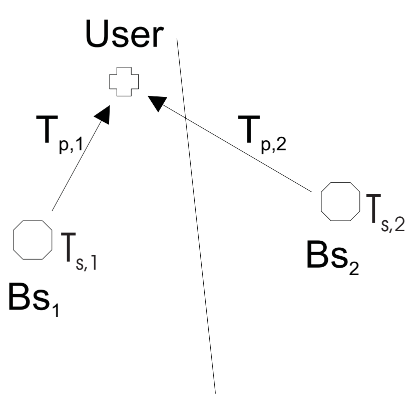

Fig. 2. Definition of $T_{s,i}$ and $T_{p,i}$

The measured time difference that a certain user can observe is given as follows:

$$\Delta T_m = T_2 - T_1, \tag{1}$$

where the times T_1 and T_2 consist of a part that is caused by the propagation time and a part that comes from the different time synchronization of the base station:

$$T_i = T_{p,i} + T_{s,i} \quad , i \in \{1, 2\}, \tag{2}$$

so that the measured time difference may be written as

$$\Delta T_m = \Delta T_p + \Delta T_s. \tag{3}$$

III. Algorithms

In this section, two different algorithms are presented that facilitate the time synchronization of the base stations participating in the network.

A. *Maximum Likelihood Estimation Based Algorithm*

With L users connected to the base station of interest, consider the maximum likelihood estimator (MLE) [3] of the base station time offset:

$$\Delta T_{s,est} = \arg\max_{\Delta T_s} p_{\Delta \mathbf{T}_m|\Delta T_s}(\Delta \mathbf{T}_m|\Delta T_s), \tag{4}$$

where

$$\Delta \mathbf{T}_m = (\Delta T_{m,1}, ..., \Delta T_{m,L})^T \tag{5}$$

is the vector containing the measured time offset from every user in the cell.

The measured time offset is a random variable since the position of the users and thus the propagation time is not known to the base station.

With the assumption that the users are independent and identically distributed (i.i.d.) in the (x, y)-plane, the probability density function of the measured time offset vector may be expressed as

$$p_{\Delta \mathbf{T}_m|\Delta T_s}(\Delta \mathbf{T}_m|\Delta T_s) = \prod_{i=1}^{L} p_{\Delta T_{m,i}|\Delta T_s}(\Delta T_{m,i}|\Delta T_s). \tag{6}$$

Eq. (6) may be rewritten using (3) as

$$p_{\Delta \mathbf{T}_m|\Delta T_s}(\Delta \mathbf{T}_m|\Delta T_s) = \prod_{i=1}^{L} p_{\Delta T_p}(\Delta T_{m,i} - \Delta T_s), \tag{7}$$

where the function $p_{\Delta T_p}(\Delta T_p)$ describes the distribution of the propagation time difference for the users associated to the regarded base station.

Since the propagation time difference is a function of the user location, this distribution depends on the user distribution in the plane. For a user at (x, y) and two base stations BS_1 and BS_2 at $(0, \pm e)$, the propagation time difference can be shown to be

$$\Delta T_p(x, y) = \frac{(\sqrt{(x^2 + (y+e)^2} - \sqrt{x^2 + (y-e)^2})}{c_0} \tag{8}$$

and c_0 is the speed of light.

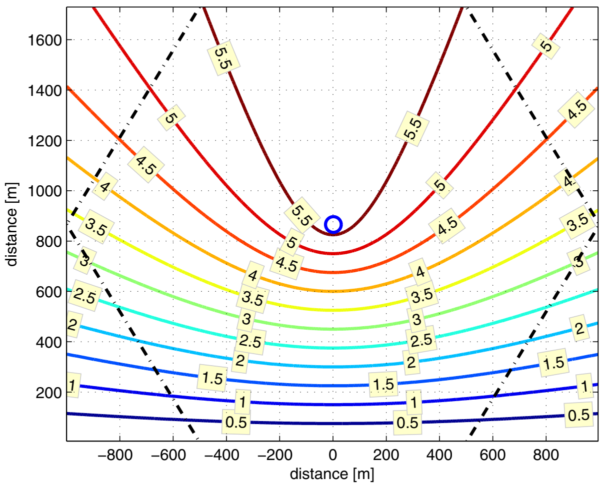

Fig. 3. Contour plot of the propagation time difference ΔT_p [μs] for base stations at $(0, \pm 866\,m)$

Figure 3 shows a contour plot of the propagation time difference for users associated to the base station at $(0, 866\,\mathrm{m})$, while the other base station is located at $(0, -866\,\mathrm{m})$.

The probability density function $p_{x,y}(x, y)$, describing the distribution of the users that are connected to the regarded base station, can be transformed to a function depending on the user position x and the propagation time difference ΔT_p according to (cf. [4]):

$$p_{x,\Delta T_p}(x, \Delta T_p) = \frac{1}{|J(x, y)|} p_{x,y}(x, y), \tag{9}$$

where $|J(x, y)|$ is the Jacobian determinant given by

$$|J(x, y)| = \begin{vmatrix} 1 & 0 \\ \frac{\partial \Delta T_p}{\partial x} & \frac{\partial \Delta T_p}{\partial y} \end{vmatrix}. \tag{10}$$

With this notation, the probability density function of the observed propagation time difference can be obtained by integration over x:

$$p_{\Delta T_p}(\Delta T_p) = \int_{-\infty}^{+\infty} p_{x,\Delta T_p}(x, \Delta T_p) dx \tag{11}$$

Even for very simple user densities such as a uniform distribution within a circle or hexagon, a closed-form solution for (11) cannot easily be obtained. However, the maximum likelihood estimator can still be derived in a closed form if we restrict our further attention to probability density functions of ΔT_p with two simple properties:

- *Property 1:* The probability density function is identical to zero for propagation time differences larger than a certain value.

$$p_{\Delta T_p}(\Delta T_p) \equiv 0 \quad \text{for} \quad \Delta T_p > \Delta T_{p,max} \tag{12}$$

- *Property 2:* The probability density function is monotonously increasing up to $\Delta T_p = \Delta T_{p,max}$.

Property 1 is fulfilled irrespective of the user distribution $p_{x,y}(x, y)$ as can be seen in (8): The maximum possible propagation time difference is observed for users located at $(0, y_0)$ with $y_0 > e$ yielding

$$\Delta T_{p,max} = \frac{2e}{c_0}. \tag{13}$$

A larger time difference is not possible, therefore the probability density function is identical to zero for time differences larger than this value.

Property 2 is fulfilled for many reasonable user distributions as shown in Section IV.

For probability density functions fulfilling Property 1 and Property 2, the solution to (4) is easily found: For a single user, the maximum likelihood estimator is given as

$$\Delta T_{s,est} = \arg\max_{\Delta T_s} p_{\Delta T_m|\Delta T_s}(\Delta T_m|\Delta T_s). \tag{14}$$

With the identity

$$p_{\Delta T_m}(\Delta T_m|\Delta T_s) = p_{\Delta T_p|\Delta T_s}(\Delta T_m - \Delta T_s|\Delta T_s), \tag{15}$$

the maximum is obtained at

$$\Delta T_{s,est} = \Delta T_m - \Delta T_{p,max}, \tag{16}$$

since $p_{\Delta T_p}(\Delta T_p)$ peaks at $\Delta T_{p,max}$.

For $L \geq 1$, the solution to (4) can be seen as follows: Consider increasing values of ΔT_s, beginning at $\Delta T_s = -\infty$. According to Property 1, $p_{\Delta \mathbf{T}_m|\Delta T_s}(\Delta \mathbf{T}_m|\Delta T_s)$ has got the first non-zero value at $\max_l\{\Delta T_{m,l}\} - \Delta T_{p,max}$.

According to Property 2, the probability function decreases for higher values than this, since every factor in the product-form of (7) decreases. Therefore, the overall maximum of the likelihood function is obtained at

$$\Delta T_{s,est} = \max_l\{\Delta T_{m,l}\} - \Delta T_{p,max}, \tag{17}$$

which is the solution to (4).

With this estimate of the time shift relative to the adjacent base stations, each base station in the network could adjust its clock according to

$$T_s(k+1) = T_s(k) + \mu \cdot \Delta T_{s,est} \tag{18}$$

with a small enough μ to allow convergence. However, with such an clock update algorithm, the clock of every base station in the network would be changed an infinite number of times even if only a single base station would have a timing error at the beginning of the synchronization procedure. Therefore we employ the update rule

$$T_s(k+1) = T_s(k) + \mu \cdot max(\Delta T_{s,est}, 0), \tag{19}$$

which avoids these oscillations and and allows for a higher value of the step-size parameter.

B. Heuristic Algorithm

The heuristic time synchronization algorithm as described in [2] constitutes an alternative approach to the method for time synchronization reported in [5]. Taking all measured time offsets $T_{m,i}$ from all users L associated to the base station into account, the base station estimates the true time offset according to the mean value of all fed back offsets:

$$\Delta T_{s,est} = \frac{1}{L} \sum_{l=1}^{L} T_{m,l} \tag{20}$$

and adjusts its time base in the next step according to

$$T_s(k+1) = T_s(k) + \mu \cdot \Delta T_{s,est} \tag{21}$$

with a small enough μ to allow convergence, just as in the case with the MLE based synchronization algorithm.

IV. Distribution of the Propagation Time Difference

In order to assess for which kinds of user distributions Property 2 is fulfilled, Equation (11) has been evaluated for a number of reasonable user distributions.

A. Uniform Distribution in a Disc

The first user distribution describes users being equiprobably located within a disc of radius $r_0 = 900\,\mathrm{m}$ around the base station at $(0,0)$, to which they are connected.

$$p_A(x,y) = \begin{cases} \frac{1}{\pi r_0^2} & , \quad x^2+y^2 < r_0^2 \\ 0 & , \quad \text{else} \end{cases} \tag{22}$$

B. Uniform Distribution in a Hexagon

The second user distribution describes users being equiprobably located within a hexagon of side length $a = 2\sqrt{3}^{-1} 866\,\mathrm{m}$ around the base station at $(0,0)$. Such a hexagon is e.g. depicted in Fig. 1.

$$p_B(x,y) = \begin{cases} \frac{2}{3\sqrt{3}a^2} & , \quad |x+\frac{y}{\sqrt{3}}| < a, |x| < a, |y| < \frac{\sqrt{3}}{2}a \\ 0 & , \quad \text{else} \end{cases} \tag{23}$$

C. Gaussian Distribution

This distribution assumes users being Gaussian distributed with a mean of $(0, 0\,\mathrm{m})$ and a standard deviation of $\sigma_0 = 866\,\mathrm{m}$. Such a distribution, where users are not necessarily connected to the base station to which they have the minimum distance, can be observed if the link to a more distant base station is stronger because of shadowing.

$$p_C(x,y) = \frac{1}{2\pi\sigma_0^2} \cdot e^{-\frac{x^2+y^2}{2\sigma_0^2}} \quad -\infty < x,y < \infty \tag{24}$$

D. Distribution Based on Signal Power

In this distribution, a user is associated to the base station, from which it receives the highest signal power. In scenarios without shadowing, this corresponds to the base station from which the user has the minimum distance. If users are assumed to be distributed uniformly in cartesian coordinates, the distribution of the users that are connected to the regarded cell at $(0,0)$ is given in IV-B. Otherwise, if the signals from different base stations have a total gain consisting of pathloss and shadowing, a different distribution is obtained. A Monte Carlo simulation was performed to determine the distribution of the users in the (x,y)-plane associated to the regarded base station in the origin. Correlated shadowing between different base stations was assumed to take situations into account, where a user is e.g. inside a building and is therefore shadowed from all base stations simultaneously. The correlation coefficient was assumed to be equal to 50%. Table I lists the assumed simulation parameters.

Parameter	**Value**
Intersite distance	1732 m
Sectorized	No
Path loss exponent	3.76
User placement	Uniform (cartesian)
Shadowing standard deviation	6 dB
Correlation of shadowing coefficient between different base stations	50%

TABLE I
DEFAULT SIMULATION PARAMETERS

The resulting probability density function of the propagation time difference is shown in Fig. 4. Please note that for the sake of better readability the different functions have been plotted at different heights. The behavior for all of the above-mentioned four user distributions is similar: The function is monotonously increasing up to a maximum value, which is obtained at $\Delta T_{p,max}$, after which all distributions are identical to zero. The peak at $\Delta T_{p,max}$ is very strong, which will be beneficial in the maximum likelihood estimation of the time difference. For these functions, both Property 1 and Property 2 are fulfilled.

The monotonous increase can also deduced from Fig. 3. The area between neighboring contour lines increases for increasing values of the propagation time difference, with the maximum area corresponding to a maximum propagation time difference of over $5.5\,\mu$s.

V. Results

For the evaluation of the synchronization algorithms, the network consisting of $N_C = 19$ cells shown in Fig. 1 has been assumed with an initial time offset of the base stations uniformly distributed within [-160 ms,160 ms]. The resulting root mean square error (RMSE), being given as

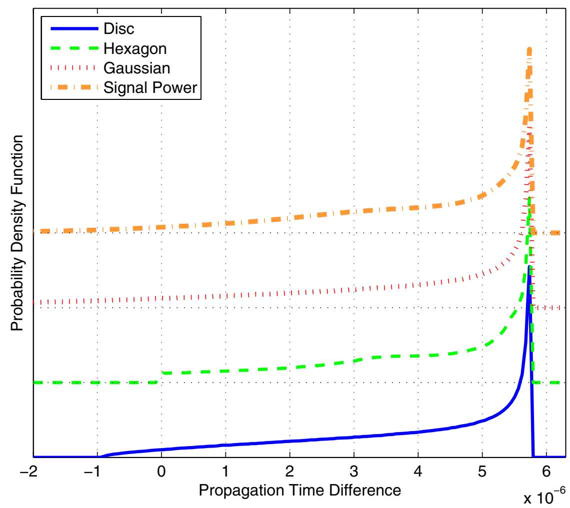

Fig. 4. Probability density function of the propagation time difference $p_{\Delta T_p}(\Delta T_p)$ for four different user distributions $p_{x,y}(x,y)$

$$e_{RMS}(k) = \sqrt{\frac{1}{N_C}\sum_{n=1}^{N_C}\left(T_{s,n}(k) - \bar{T}_s(k)\right)^2}, \qquad (25)$$

with

$$\bar{T}_s(k) = \frac{1}{N_C}\sum_{n=1}^{N_C} T_{s,n}(k) \qquad (26)$$

of the base station time offsets as a function of the iteration number is shown in Fig. 5.

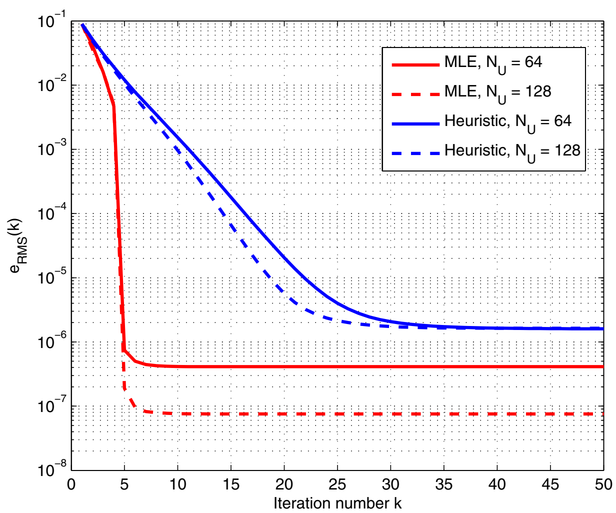

Fig. 5. Root mean square error $e_{RMS}(k)$[s] of the base stations in the network, intersite distance 1732 m

For a total number of users in the network of $N_U = 64$, the heuristic synchronization algorithm achieves steady-state performance after about 30 iterations. Here, a root mean square error in the order of $e_{RMS} = 1.5\,\mu s$ is obtained. The synchronization algorithm based on the maximum likelihood estimation of the propagation time difference converges significantly faster: After about seven iterations, the steady-state is reached, achieving an accuracy of about $e_{RMS} = 0.4\,\mu$s. Clearly, while such an accuracy can only be achieved with the above-mentioned assumptions of direct-path propagation and error-free estimation of the propagation time difference ΔT_p in the mobiles, the proposed maximum-likelihood based time synchronization algorithm performs significantly better than the heuristic one. For a number of $N_U = 128$ users in the network, the performance of both algorithms increases, but in different ways: For the heuristic algorithm, convergence is faster and the steady-state is reached after about 25 iterations, but the resulting accuracy is not increased significantly. For the maximum likelihood estimation based algorithm, the number of iterations needed for convergence is not reduced, but the overall accuracy is increased to a RMSE of below $e_{RMS} = 0.1\,\mu$s.

VI. Conclusion

The necessity for the time synchronization between base stations participating in single frequency network operation has been pointed out, motivating the need for base station time synchronization algorithms. A novel network synchronization algorithm based on the maximum likelihood estimation of the intersite time difference in the mobile users of a cellular network has been presented. The performance of this algorithm has been evaluated and compared to a previously published, heuristic approach, showing that the MLE based algorithm significantly outperforms the heuristic one. The improved performance is achieved by exploiting only the knowledge of the propagation time, or alternately the distance, from one base station to the other. Such knowledge seems to be obtainable with little effort.

The sensitivity to time difference measurement errors has not been investigated in this paper. Since the MLE based algorithm depends only on a single measurement, its sensitivity to imperfect time difference knowledge will be higher than in the case of the heuristic algorithm. Performance in the case of time difference measurement errors will be studied in future work.

References

[1] 3GPP, "Multimedia Broadcast/Multicast Service (MBMS); Architecture and functional description," TS 23.246 v7.0.0.

[2] E. Costa, P. Slanina, V. Bochnicka, E. Schulz, G. Spano Greco, "Downlink based Intra- and Inter-Cell Time and Frequency Synchronization in OFDM Cellular Systems," *6th World Wireless Congress WWC* 2005.

[3] S. Kay, *Fundamentals of Statistical Signal Processing: Estimation Theory*, Prentice Hall Signal Processing Series, Upper Saddle River, New Jersey 1993.

[4] A. Papoulis, S. U. Pillai, *Probability, Random Variables and Stochastic Processes*, McGraw Hill, New York 2002.

[5] D. Galda, N. Meier, H. Rohling, M. Weckerle, "System Concept for a Self-Organized Cellular Single Frequency OFDM Network," in Proceedings of the 8^{th} *Int. OFDM Workshop 2003*, Hamburg, Sep. 2003.