Modeling and Performance Evaluation for Mobile Ricean MIMO Channels

Adrian Ispas*, Jonas Hölscher*, Xitao Gong*, Christian Schneider†, Gerd Ascheid*, and Reiner Thomä†

*Chair for Integrated Signal Processing Systems, RWTH Aachen University, Germany

{ispas, gong, ascheid}@iss.rwth-aachen.de, jonas.hoelscher@intel.com

†Institute of Information Technology, Ilmenau University of Technology, Germany

{christian.schneider, reiner.thomae}@tu-ilmenau.de

Abstract—We propose a general channel model for mobile Ricean multiple-input multiple-output (MIMO) channels, which is characterized by parameters that are readily obtainable from measurements. To this end, a moment-based channel decomposition is derived. For the case of statistical channel state information (CSI) at the transmitter (TX) and instantaneous CSI at the receiver, we derive an approximation of the achievable rate, i.e., the mutual information (MI), which is only a function of the channel parameters of the proposed channel model and thus gives insight into the parameters that influence the MI. Finally, we evaluate the MI for a 4×4 MIMO system based on channel measurements at 2.53 GHz for two low-complexity TX strategies, i.e., a beamforming (BF) and a spatial multiplexing (SM) strategy. We find that the proposed channel model and the approximate evaluation of the MI are both accurate for realistic signal-to-noise ratio (SNR) values. The approximate evaluation is able to reproduce crossing points between the MI of the considered TX strategies; it thus reflects the SNR at which one should switch from the BF to the SM strategy.

I. Introduction

Multiple-input multiple-output (MIMO) transmission is by now a well established method to enhance the spectral efficiency over wireless channels. In general, the usefulness of MIMO techniques depends on the channel knowledge at the transmitter (TX) and the receiver (RX), the signal-to-noise ratio (SNR), and the channel conditions [1], [2]. In order to understand the influence of the channel conditions on the spectral efficiency of MIMO techniques, the use of appropriate channel models is crucial. Unfortunately, an accurate and reasonably simple modeling of realistic MIMO channels at the same time is a difficult task [3]. One usually resorts to several restrictive assumptions, e.g., Rayleigh fading, the Kronecker model, or even one-sided correlation only, to assess the influence of the channel parameters on the spectral efficiency.

Analytically tractable and accurate MIMO channel models are usually provided by correlation-based analytical models [3]. When a dominant component is present, a deterministic part is added to the stochastic channel matrix [4], [5]. This yields a classical Ricean channel model, where the deterministic part is usually linked to a line-of-sight (LOS) contribution and modeled by a rank-1 matrix [4], [6]. Such a model is relevant for fixed scenarios without any movement of the TX and the RX. In a mobile setting, the dominant component

is not deterministic anymore and thus more challenging to model; this is an issue that has remained mostly unexplored until now. Furthermore, the achievable rate over such channels is dependent on the statistical parameters of the channel model, but their influence is not well understood.

Contributions: We study the modeling and evaluate the achievable rate of mobile Ricean MIMO channels in a setting where the TX has only statistical channel state information (CSI), while the RX has instantaneous CSI. Exemplarily, we focus on the low-complexity TX strategies beamforming (BF) and spatial multiplexing (SM). We make the following contributions:

- We propose a general channel model for mobile Ricean MIMO channels. Correspondingly, we derive a moment-based channel decomposition, which gives the model parameters from measured data.
- We give an approximate evaluation of the achievable rate over the proposed channel model in terms of the mutual information (MI). The approximate evaluation is only a function of the statistical parameters of the proposed channel model; it thus gives insight into the channel parameters influencing the achievable rate.
- We evaluate the achievable rate for a 4×4 MIMO setup based on urban macrocell measurements at 2.53 GHz. Furthermore, the accuracy of the channel modeling approach and the approximate evaluation of the MI are confirmed for realistic SNR values. The approximate evaluation of the MI is able to reproduce the crossing points between the MI of the two TX strategies, i.e., the SNR at which one should switch from BF to SM [7].

Notation: $\mathbf{A}^*$, $\mathbf{A}^T$, and $\mathbf{A}^H$ denote the (element-wise) complex conjugate, the transpose, and the conjugate transpose of the matrix $\mathbf{A}$, respectively. The unique Hermitian positive semidefinite square root of a Hermitian positive semidefinite matrix $\mathbf{A}$ is represented by $\mathbf{A}^{\frac{1}{2}}$. $||\mathbf{A}||_F$, $\mathrm{tr}\{\mathbf{A}\}$, and $\lambda_{\max}(\mathbf{A})$ denote the Frobenius norm, the trace, and the maximal eigenvalue of the matrix $\mathbf{A}$, respectively. For two matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \odot \mathbf{B}$ is the Hadamard (element-wise) product. The vectorization, i.e., the column-wise stacking, of the matrix $\mathbf{A}$ is denoted by $\mathrm{vec}\{\mathbf{A}\}$. The $N \times N$ identity matrix is represented by $\mathbf{I}_N$ and the $MN \times MN$ commutation matrix $\mathbf{K}_{M,N}$ fulfills $\mathbf{K}_{M,N}\,\mathrm{vec}\{\mathbf{A}\} = \mathrm{vec}\{\mathbf{A}^T\}$ for an $M \times N$ matrix $\mathbf{A}$. $[\mathbf{A}]_{k,l}$ denotes the element in the k-th row and the l-th column of $\mathbf{A}$.

The expectation of a random variable x is denoted by $\mathrm{E}\{x\}$. $\log(\cdot)$ is the logarithm to the base 2, and $\ln(\cdot)$ is the natural logarithm. The imaginary unit is represented by j.

II. System Model

We consider transmission over a time-varying and frequency-flat fading MIMO channel with N_{TX} antennas at the TX and N_{RX} antennas at the RX. The input-output relation is given by the received signal

$$\mathbf{y}[m] = \mathbf{H}[m]\mathbf{x}[m] + \mathbf{n}[m] \tag{1}$$

for the time slot $m \in \mathbb{Z}$. The random channel weights $\mathbf{H}[m]$ of size $N_{\mathrm{RX}} \times N_{\mathrm{TX}}$ are jointly proper. The zero-mean samples $\mathbf{x}[m]$ are the jointly proper Gaussian transmitted vectors of length N_{TX} that are uncorrelated in time with covariance matrix $\mathrm{E}\{\mathbf{x}[m]\mathbf{x}^H[m]\} = P_x\mathbf{Q}[m]$ and $\mathrm{tr}\{\mathbf{Q}[m]\} = 1$. The samples $\mathbf{n}[m]$ denote white jointly proper Gaussian noise vectors in time and space of length N_{RX} with covariance matrix $\mathrm{E}\{\mathbf{n}[m]\mathbf{n}^H[m]\} = \sigma_n^2\mathbf{I}_{N_{\mathrm{RX}}}$. We define the (nominal) SNR as $\rho = P_x/\sigma_n^2$. Furthermore, we assume the RX to have CSI, i.e., the RX knows the current channel realization $\mathbf{H}[m]$.

We compare two different TX techniques. The first technique is SM with the covariance matrix of the input given by $P_x\mathbf{Q}_{\mathrm{SM}}[m] = (P_x/N_{\mathrm{TX}})\mathbf{I}_{N_{\mathrm{TX}}}$. In this case, no CSI is needed at the TX. The second technique is statistical BF with the covariance matrix of the input $P_x\mathbf{Q}_{\mathrm{BF}}[m] = P_x\mathbf{u}_{\mathbf{R}_{\mathrm{TX}},\max}^*[m]\mathbf{u}_{\mathbf{R}_{\mathrm{TX}},\max}^T[m]$ where $\mathbf{u}_{\mathbf{R}_{\mathrm{TX}},\max}[m]$ is the dominant eigenvector of the TX correlation matrix of the channel $\mathbf{R}_{\mathrm{TX}}[m] = \mathrm{E}\{\mathbf{H}^T[m]\mathbf{H}^*[m]\}$[1]. Here, only (statistical) knowledge in the form of $\mathbf{u}_{\mathbf{R}_{\mathrm{TX}},\max}[m]$ is required. Note that in both cases the same TX power P_x is used, since $\mathrm{tr}\{\mathbf{Q}_{\mathrm{SM}}[m]\} = \mathrm{tr}\{\mathbf{Q}_{\mathrm{BF}}[m]\} = 1$ holds.

III. Channel Modeling and Decomposition

In this section, we describe the channel modeling according to a correlation-based analytical model [3], which is based on a multivariate proper Gaussian distribution. These models only require statistical parameters that are, in general, readily available from measured data, and they are analytically tractable. The Gaussian distribution of the channel can be justified by the occurrence of a large number of independent scatterers. If an additional dominant component is present, the channel amplitude is usually characterized by a Ricean distribution. The ratio between the power of the dominant component and the power of the remaining weaker components is then described by the K-factor. Often the dominant component corresponds to a LOS contribution; however, we note that this does not always have to be the case [8], e.g., the popular K-factor estimator [9] based on a moment method does not make any LOS assumption.

In a fixed scenario without any movements of the TX and the RX, a LOS component results in a Gaussian distribution of the channel with a non-zero mean at every single frequency. However, if we consider a mobile terminal (MT) or treat the channel at different frequencies inside a stationarity region as

distinct channel realizations, the channel component due to a strong scatterer or LOS exhibits a varying phase and thus the mean of the channel is zero [10]. Based on the above considerations, we propose the following channel model:

$$\mathbf{H}[m] = \mathbf{V}[m] \odot \mathbf{\Phi}[m] + \tilde{\mathbf{H}}[m] \tag{2}$$

where $\mathbf{V}[m]$ contains the deterministic amplitudes of the dominant components and $\mathbf{\Phi}[m]$ contains the corresponding random phases, i.e., $[\mathbf{\Phi}[m]]_{k,l} = e^{j\phi_{(l-1)N_{\mathrm{RX}}+k}[m]}$, $k = 1, \ldots, N_{\mathrm{RX}}$, $l = 1, \ldots, N_{\mathrm{TX}}$. The matrix $\tilde{\mathbf{H}}[m]$ denotes the random part of the channel due to the remaining (weaker) scatterers; it is modeled by a zero-mean jointly proper Gaussian matrix[2]. The main difficulty in the modeling and characterization of the channel lies in the phases $\phi_p[m]$, $p = 1, \ldots, N_{\mathrm{TX}}N_{\mathrm{RX}}$ of the MIMO sub-links, since they characterize the correlations between the dominant components. This has already been highlighted in [11]. We thus make the following assumptions for the MIMO sub-links denoted by $p, q = 1, \ldots, N_{\mathrm{TX}}N_{\mathrm{RX}}$:

1) $\phi_p[m]$ is independent of $\tilde{\mathbf{H}}[m]$,
2) $\phi_p[m]$ is uniformly distributed over $[-\pi, \pi)$,
3) $\Delta_\phi^{p,q}[m] = \phi_p[m] - \phi_q[m]$ is deterministic.

The first two assumptions are self-explanatory [11]; however, a note is in order regarding the last assumption. As mentioned above, the contributions from the dominant components are not deterministic due to the mobility of the MT and the consideration of different frequencies. For the case that all MIMO sub-links are influenced by the same dominant component and the distance between the TX and the RX is considerably larger than the array sizes, the resulting phase shifts, i.e., the changes in the phases, can be modeled as equal for all sub-links. Therefore, the $\Delta_\phi^{p,q}[m]$ are modeled as constant inside a region of constant statistics, i.e., they are deterministic. Obviously, assumption 3) is not fulfilled for all antenna setups, e.g., it would not necessarily hold for a MIMO system made of directional antennas with different orientations.

A. Moment-Based Channel Decomposition

We now present a method to separate the contribution of the dominant component from the remaining part of the channel and thus to obtain the statistical parameters of (2). The random phases $\phi_p[m]$ of the dominant component are problematic in our attempt to model and generate the channel using parameters extracted from measured data. The reason is that we cannot simply take the mean of the measurements to separate the statistical channel parameters of the dominant and the remaining components. We are thus striving for a simple method based on the proposed channel model, which yields the decomposition without using more involved multipath component estimation techniques. Our method is based on the evaluation of certain moments of the channel, and it is similar in spirit to the K-factor estimation in [9]. While in [9] only the amplitude of the channel is considered, we also account for the phase of the channel to obtain the channel decomposition.

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[1]The eigenvectors of $\mathbf{R}_{\mathrm{TX}}^*[m]$ form the optimal precoding with respect to the Jensen bound on the MI [1].

[2]In order to simplify the presentation, we do not introduce a K-factor at this point; instead the power adjustement due to the K-factor is contained in $\mathbf{V}[m]$ and $\tilde{\mathbf{H}}[m]$.

In order to simplify notation, we drop the time argument for the remainder of this section, and, for $p, q = 1, \ldots, N_{\text{TX}}N_{\text{RX}}$, we define the variables $h_p = [\text{vec}\{\mathbf{H}\}]_{p,1}$, $V_p = [\text{vec}\{\mathbf{V}\}]_{p,1}$, and $\tilde{h}_p = [\text{vec}\{\tilde{\mathbf{H}}\}]_{p,1}$. Using $h_p = V_p e^{j\phi_p} + \tilde{h}_p$ and the identity [12]

$$\text{E}\left\{\tilde{h}_p^*\tilde{h}_q^*\tilde{h}_k\tilde{h}_l\right\} = \text{E}\left\{\tilde{h}_p^*\tilde{h}_k\right\}\text{E}\left\{\tilde{h}_q^*\tilde{h}_l\right\} + \text{E}\left\{\tilde{h}_q^*\tilde{h}_k\right\}\text{E}\left\{\tilde{h}_p^*\tilde{h}_l\right\}$$

we can obtain the following moments:

$$r_{p,q} = \text{E}\left\{h_p h_q^*\right\} = V_p V_q z_{p,q} + \tilde{r}_{p,q} \quad (3)$$

$$s_{p,q} = \text{E}\left\{h_p h_q^* h_p^* h_p\right\} = V_p^2(r_{p,q} + \tilde{r}_{p,q}) + 2\tilde{r}_{p,p} r_{p,q} \quad (4)$$

with $\tilde{r}_{p,q} = \text{E}\left\{\tilde{h}_p\tilde{h}_q^*\right\}$ and $z_{p,q} = \text{E}\left\{e^{j\Delta_\phi^{p,q}}\right\}$. The proof is omitted due to space constraints. Furthermore, with (3) and (4), it can be shown that

$$V_p V_q z_{p,q} = r_{p,q} - \tilde{r}_{p,q} = \frac{2r_{p,p}r_{p,q} - s_{p,q}}{V_p^2}. \quad (5)$$

With (2), we obtain

$$\mathbf{R} = \text{E}\left\{\text{vec}\{\mathbf{H}\}\left(\text{vec}\{\mathbf{H}\}\right)^H\right\}$$
$$= \left(\text{vec}\{\mathbf{V}\}\left(\text{vec}\{\mathbf{V}\}\right)^H\right)\odot\boldsymbol{\Delta}_\Phi + \tilde{\mathbf{R}} = \bar{\mathbf{R}} + \tilde{\mathbf{R}} \quad (6)$$

where we defined $[\boldsymbol{\Delta}_\Phi]_{p,q} = \text{E}\left\{e^{j\Delta_\phi^{p,q}}\right\}$ and $[\tilde{\mathbf{R}}]_{p,q} = \tilde{r}_{p,q}$. From (5), we obtain $V_p^4 = 2r_{p,p}^2 - s_{p,p}$, which yields

$$\left[\bar{\mathbf{R}}\right]_{p,q} = V_p V_q z_{p,q} = \frac{2r_{p,p}r_{p,q} - s_{p,q}}{\sqrt{2r_{p,p}^2 - s_{p,p}}}. \quad (7)$$

Using assumption 3), i.e., assuming that $\Delta_\phi^{p,q}$ is deterministic, we can rewrite (6) as

$$\mathbf{R} = \bar{\mathbf{R}} + \tilde{\mathbf{R}} = \text{vec}\{\bar{\mathbf{H}}\}\left(\text{vec}\{\bar{\mathbf{H}}\}\right)^H + \tilde{\mathbf{R}} \quad (8)$$

with $\bar{\mathbf{H}} = \mathbf{V}\odot\check{\boldsymbol{\Delta}}_\Phi$ and $[\check{\boldsymbol{\Delta}}_\Phi]_{k,l} = e^{j(\phi_{(l-1)N_{\text{RX}}+k}-\phi_1)}$, $k = 1, \ldots, N_{\text{RX}}$, $l = 1, \ldots, N_{\text{TX}}$. It immediately follows that the correlation matrix of the dominant scatterer $\bar{\mathbf{R}}$ has rank 1.

When applying this method to measured data, we need to post-process the estimate of $\bar{\mathbf{R}}$. In case $2r_{p,p}^2 \leq s_{p,p}$, we set the column and row p of the estimate to zero, and we obtain the new estimate $\check{\mathbf{R}}$. In order to make sure that the final estimate is Hermitian, positive semidefinite, and of rank 1, see (8), we first extract its Hermitian part and then only keep the contribution of the largest eigenvalue, i.e., we use

$$\hat{\bar{\mathbf{R}}} = c\,\mathbf{u}_{\check{\mathbf{R}},\max}\mathbf{u}_{\check{\mathbf{R}},\max}^H\,; \qquad \text{vec}\{\bar{\mathbf{H}}\} = \sqrt{c}\,\mathbf{u}_{\check{\mathbf{R}},\max} \quad (9)$$

where $\mathbf{u}_{\check{\mathbf{R}},\max}$ denotes the eigenvector of $\left(\check{\mathbf{R}} + \check{\mathbf{R}}^H\right)/2$ corresponding to the largest non-negative eigenvalue. Defining the estimates of $\mathbf{R}$ and $\tilde{\mathbf{R}}$ as $\hat{\mathbf{R}}$ and $\hat{\tilde{\mathbf{R}}}$, respectively, the constant c is chosen such that the positive semidefiniteness of $\hat{\tilde{\mathbf{R}}} = \hat{\mathbf{R}} - \hat{\bar{\mathbf{R}}}$ is ensured as well [13, Theorem 7.7.7]:

$$c = \min\left\{\lambda_{\max}^+\left(\frac{\check{\mathbf{R}} + \check{\mathbf{R}}^H}{2}\right), \left(\mathbf{u}_{\check{\mathbf{R}},\max}^H\hat{\mathbf{R}}^{-1}\mathbf{u}_{\check{\mathbf{R}},\max}\right)^{-1}\right\} \quad (10)$$

with $\lambda_{\max}^+(\mathbf{A}) = \max\{\lambda_{\max}(\mathbf{A}), 0\}$. Note that some power of the dominant scatterer is transferred to $\hat{\tilde{\mathbf{R}}}$ whenever $c < \lambda_{\max}(\check{\mathbf{R}} + \check{\mathbf{R}}^H)/2$. This can occur when the estimates of (3) and (4) are inaccurate.

B. Channel Generation

We can now generate channel realizations based on the statistical channel parameters according to

$$\text{vec}\{\mathbf{H}\} = \text{vec}\{\bar{\mathbf{H}}\}e^{j\phi} + \tilde{\mathbf{R}}^{\frac{1}{2}}\mathbf{g} \quad (11)$$

where ϕ is uniformly distributed over $[-\pi, \pi)$, and $\mathbf{g}$ is a zero-mean proper Gaussian random vector of length $N_{\text{TX}}N_{\text{RX}}$ with covariance matrix $\mathbf{I}_{N_{\text{TX}}N_{\text{RX}}}$; ϕ and $\mathbf{g}$ are mutually independent.

IV. Performance Evaluation

With respect to the system model in Section II, the MI between the input $\mathbf{x}[m]$ and the output $\mathbf{y}[m]$ combined with CSI at the receiver is given by

$$I(\mathbf{x}[m]; \mathbf{y}[m], \mathbf{H}[m])$$
$$= \text{E}\left\{\log\det\left(\mathbf{I}_{N_{\text{RX}}} + \rho\mathbf{H}[m]\mathbf{Q}[m]\mathbf{H}^H[m]\right)\right\}. \quad (12)$$

Note that the MI in (12) is time-dependent as the channel is in general non-stationary; therefore, in a strict sense, (12) is not an achievable rate. Nevertheless, we use the MI (12) as performance measure, since it has an interpretation in terms of an achievable rate in bit/channel use (bit/c.u.) for non-stationary slow- and fast-fading wireless channels [14], [15]. We note that the MI is invariant to the distribution of the random scalar phase of the dominant scatterer ϕ in (11). Thus, regarding the MI, the mobile Ricean channel model described in (11) and the classical Ricean channel model for fixed scenarios are equivalent.

Based on the second-order approximation of $f(\mathbf{A}) = \ln\det\mathbf{A}$ at $\text{E}\{\mathbf{A}\}$ and results from complex-valued matrix differentiation, it can be shown that the following approximate evaluation of (12) holds:

$$I(\mathbf{x}[m]; \mathbf{y}[m], \mathbf{H}[m]) \approx \log\det\left(\mathbf{I}_{N_{\text{TX}}} + \rho\mathbf{R}_{\text{TX}}^*[m]\mathbf{Q}[m]\right)$$
$$- \frac{\log(e)\rho^2}{2}\text{tr}\left\{\left(\mathbf{Q}^T[m]\otimes\mathbf{I}_{N_{\text{TX}}}\right)\mathbf{Z}[m](\mathbf{Q}[m]\otimes\mathbf{I}_{N_{\text{TX}}})\mathbf{W}[m]\right\}$$
$$(13)$$

with the $N_{\text{TX}}^2 \times N_{\text{TX}}^2$ matrices

$$\mathbf{W}[m] = \mathbf{K}_{N_{\text{TX}},N_{\text{TX}}}\left(\mathbf{T}^{-T}[m]\otimes\mathbf{T}^{-1}[m]\right) \quad (14)$$

$$\mathbf{Z}[m] = \text{E}\left\{(\mathbf{p}[m] - \mathbf{p}_0[m])(\mathbf{p}[m] - \mathbf{p}_0[m])^T\right\} \quad (15)$$

the vectors $\mathbf{p}[m] = \text{vec}\{\mathbf{H}^H[m]\mathbf{H}[m]\}$ and $\mathbf{p}_0[m] = \text{vec}\{\mathbf{R}_{\text{TX}}^*[m]\}$ of length N_{TX}^2, and the $N_{\text{TX}} \times N_{\text{TX}}$ matrix $\mathbf{T}[m] = \mathbf{I}_{N_{\text{TX}}} + \rho\mathbf{R}_{\text{TX}}^*[m]\mathbf{Q}[m]$. The proof is omitted due to space constraints.

Unfortunately, (13) requires the evaluation of second- and fourth-order moments of the channel $\mathbf{H}[m]$, i.e., of $\mathbf{R}_{\text{TX}}[m]$ and $\mathbf{Z}[m]$. In an attempt to get more insight on the channel parameters that influence the MI, we rewrite (15) as a function of $\bar{\mathbf{H}}[m]$ and $\tilde{\mathbf{R}}[m]$ only:

$$\mathbf{Z}[m] = \text{E}\left\{\tilde{\mathbf{p}}[m]\tilde{\mathbf{p}}^T[m]\right\} + \mathbf{P}_\times[m] + \mathbf{P}_\times^T[m]$$
$$- \text{vec}\left\{\tilde{\mathbf{R}}_{\text{TX}}^*[m]\right\}\left(\text{vec}\left\{\tilde{\mathbf{R}}_{\text{TX}}^*[m]\right\}\right)^T \quad (16)$$

with $\tilde{\mathbf{p}}[m] = \text{vec}\left\{\tilde{\mathbf{H}}^H[m]\tilde{\mathbf{H}}[m]\right\}$,

$$\left[\text{E}\left\{\tilde{\mathbf{p}}[m]\tilde{\mathbf{p}}^T[m]\right\}\right]_{(k-1)N_{\text{TX}}+l,(m-1)N_{\text{TX}}+n}$$
$$= \left[\tilde{\mathbf{R}}_{\text{TX}}[m]\right]_{k,l}\left[\tilde{\mathbf{R}}_{\text{TX}}[m]\right]_{m,n}$$
$$+ \sum_{q=1}^{N_{\text{RX}}}\sum_{r=1}^{N_{\text{RX}}}\left[\tilde{\mathbf{R}}[m]\right]_{(k-1)N_{\text{RX}}+q,(n-1)N_{\text{RX}}+r}$$
$$\times \left[\tilde{\mathbf{R}}[m]\right]_{(m-1)N_{\text{RX}}+r,(l-1)N_{\text{RX}}+q} \qquad (17)$$

$$\mathbf{P}_\times[m] = \left(\mathbf{I}_{N_{\text{TX}}}\otimes\bar{\mathbf{H}}^H[m]\right)\tilde{\mathbf{R}}[m]\left(\mathbf{I}_{N_{\text{TX}}}\otimes\bar{\mathbf{H}}[m]\right)\mathbf{K}_{N_{\text{TX}},N_{\text{TX}}} \quad (18)$$

and $\tilde{\mathbf{R}}_{\text{TX}}[m] = \text{E}\{\tilde{\mathbf{H}}^T[m]\tilde{\mathbf{H}}^*[m]\}$, which is directly obtainable from $\tilde{\mathbf{R}}[m]$. Due to space constraints, the proof is omitted. Note that all the necessary channel parameters are readily available from the channel decomposition in Section III-A.

In order to show the insight the proposed channel model in conjunction with the approximate evaluation of the MI can offer, we consider an example, for which we drop the time argument. Consider the change in mutual information with the power ratio of the dominant component to the remaining scatterer components, i.e., the MIMO K-factor $K = \text{tr}\left\{\bar{\mathbf{R}}\right\}/\text{tr}\left\{\tilde{\mathbf{R}}\right\} = \text{tr}\left\{\bar{\mathbf{R}}_{\text{TX}}\right\}/\text{tr}\left\{\tilde{\mathbf{R}}_{\text{TX}}\right\}$. We can now state

$$\mathbf{R}_{\text{TX}} = \frac{K}{1+K}\bar{\mathbf{R}}_{\text{TX,norm}} + \frac{1}{1+K}\tilde{\mathbf{R}}_{\text{TX,norm}} \qquad (19)$$

with the normalized correlation matrices $\bar{\mathbf{R}}_{\text{TX,norm}} = \frac{1+K}{K}\bar{\mathbf{R}}_{\text{TX}}$ and $\tilde{\mathbf{R}}_{\text{TX,norm}} = (1+K)\tilde{\mathbf{R}}_{\text{TX}}$. For simplicity, we only consider the first term in (13), i.e., the Jensen bound on the MI. It is straightforward to show that, for fixed $\bar{\mathbf{R}}_{\text{TX,norm}}$ and $\tilde{\mathbf{R}}_{\text{TX,norm}}$, the derivative of the Jensen bound on the MI with respect to K is greater or equal to 0, i.e., the Jensen bound on the MI increases with increasing K if and only if

$$\text{tr}\left\{\left(\mathbf{I}_{N_{\text{TX}}}+\rho\mathbf{R}_{\text{TX}}^*\mathbf{Q}\right)^{-1}\left(\bar{\mathbf{R}}_{\text{TX,norm}}^* - \tilde{\mathbf{R}}_{\text{TX,norm}}^*\right)\mathbf{Q}\right\} \geq 0 \quad (20)$$

holds. Otherwise, the Jensen bound on the MI is strictly decreasing with an increasing K. In the case of BF, (20) can be simplified to the SNR-independent inequality

$$\mathbf{u}_{\mathbf{R}_{\text{TX}},\max}^H\bar{\mathbf{R}}_{\text{TX,norm}}\mathbf{u}_{\mathbf{R}_{\text{TX}},\max} \geq \lambda_{\max}\left(\mathbf{R}_{\text{TX}}\right). \qquad (21)$$

V. CHANNEL MEASUREMENTS

The channel measurements were performed at 2.53 GHz in two bands of 45 MHz in Ilmenau, Germany, which corresponds to a scenario of urban macrocell type. The measurement campaign sequentially covered three base station (BS) positions with different heights and a multitude of MT tracks for vertical and horizontal polarizations. The MT is a car driving with a maximal velocity of about 10 km/h. In this paper, the three BS positions at a height of 25 m and the three MT reference tracks are selected, see [16] for more details. Furthermore, we use a 20 MHz band centered at 2.505 GHz for the evaluations. We post-process the measurement data by estimating a noise level in the time-delay domain and not considering any values below it. In order to estimate the statistical quantities, we approximate the ensemble averaging by an averaging in time over $N_t = 16$ and in frequency over $N_f = 128$ samples. The resulting total of 2048 (≈ 500

	BS	Track	MT orient.	MT pos. [m]	K-Factors
Link 1	1	41a-42	back	$0-34.9$	low
Link 2	3	41a-42	back	$0-31.5$	low
Link 3	3	9a-9b	left	$0-38.9$	medium
Link 4	2	10b-9a	front	$9.8-56.8$	high
Link 5	3	10b-9a	left	$0-64.9$	varying

non-coherent) realizations are a reasonable choice to estimate the statistics of the channel [16]. We normalize the channel such that $\text{E}\left\{||\mathbf{H}[m]||_F^2\right\} = N_{\text{TX}}N_{\text{RX}}$ is fulfilled inside each stationarity region of size N_t in time and N_f in frequency.

At the BS (TX) we have a uniform linear array, and at the MT (RX) we use two superimposed uniform circular arrays (UCAs). The elements at the MT correspond to the front (direction of motion), the back, and the two sides of the MT. We consider a 4×4 vertically polarized MIMO setup. The antennas are separated by λ_c at the BS and $0.5\lambda_c$ (different UCAs) or $0.327\lambda_c$ (same UCA) at the MT, where λ_c is the carrier wavelength. See [17] for details on the antennas.

A. Scenario Classification

Based on the measurements, we mainly observe links with either low K-factors and low MIMO sub-link correlations or links with high K-factors and high correlations. A similar observation was made in [18]. Thus, similar to [19], we classify the measurements in links with low, medium, and high K-factors. For the links with low K-factors, mostly values in $[0,2]$ are observed. The medium and high K-factors links have several peaks with values above 5 and 10, respectively. Additionally, we specify one link with varying K-factors, i.e., it consists of segments of low and high K-factors. The chosen links are specified in Table I. The reason for the low K-factors/correlations in link 1 and 2 is that track 41a-42 is partly located in a street canyon; regarding BS 1 and 3 no dominant components are expected. In contrast, tracks 9a-9b and 10b-9a are mostly situated in an open environment where dominant components are more likely to occur.

VI. RESULTS

First, we investigate the accuracy of the proposed channel model with the corresponding decomposition. As can be seen from Table II, we obtain a good accuracy in terms of the root mean square error (RMSE) between the MI of the modeled channel (11) and the measured channel. Only at high SNR with SM, the inaccuracy tends to increase.

Now, we compare the approximate evaluation of the MI, i.e., (13) with (14) and (16), to the MI (12) of the measured channel for the links 1-4. The results are accumulated over each track and shown as a function of the SNR in Fig. 1. We see that the crossing points between the MI of BF and SM lie in different SNR ranges depending on the link. The crossing point is between 0 and 5 dB for the links with low K-factors and between 10 and 15 dB for the links with medium/high K-factors. The approximate evaluation is able to reproduce the crossing points inside these domains.

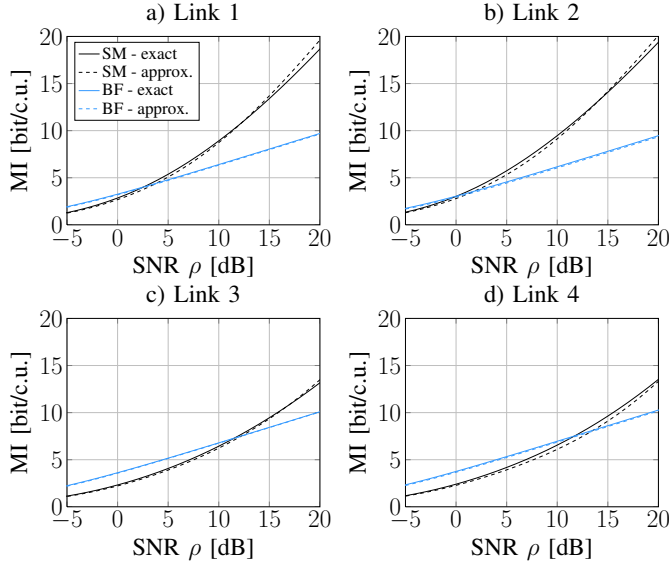|	RMSE of the MI of the modeled channel [bit/c.u.]					
	SM			BF		
SNR ρ [dB]	0	10	20	0	10	20
Link 1	0.051	0.121	0.158	0.064	0.079	0.081
Link 2	0.091	0.282	0.445	0.100	0.132	0.137
Link 3	0.070	0.156	0.232	0.110	0.140	0.144
Link 4	0.070	0.145	0.182	0.115	0.143	0.147

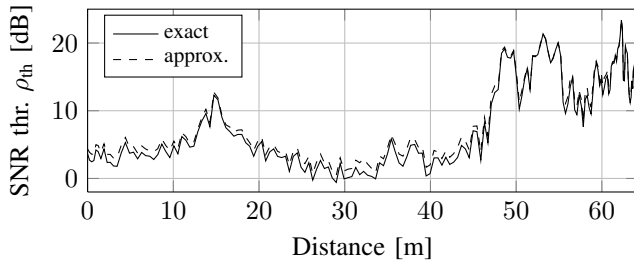Fig. 1. MI vs. SNR of the measured channel and the approximate evaluation.

Fig. 2. SNR threshold between BF and SM vs. distance for the exact and the approximate evaluation on link 5.

Finally, in Fig. 2, we show the evolution of the SNR threshold ρ_{th} at which SM starts to have a higher MI than BF for link 5. This link is characterized by high K-factors at the beginning and at the end of the link only, and thus the SNR threshold is higher in these domains. The reason is the presence of a street canyon in the middle of the track, which shadows a strong scatterer. The approximate evaluation of the MI shows a good agreement with the exact evaluation.

VII. CONCLUSION

In this paper, we proposed a general channel model for mobile Ricean MIMO channels along with a moment-based channel decomposition to obtain the corresponding parameters. Furthermore, an approximate evaluation of the MI using the parameters of the channel model was derived and used to assess the performance of the two low-complexity TX strategies BF and SM. Using urban macrocell channel measurements at 2.53 GHz, we find that the proposed channel model and the approximate evaluation of the MI using the proposed channel model are both accurate for a 4×4 MIMO setup at realistic SNR values. The approximate evaluation is able to reproduce crossing points between the MI of the considered TX strategies.

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