Analysis of Mismatched Downlink Beamforming over Non-Stationary Channels with Interference

(Invited Paper)

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Abstract

Inter-cell interference is the limiting factor in next generation cellular networks with frequency reuse factor 1. A promising approach to mitigate the effects of downlink interference is the use of transmit beamforming; however, only few measurement-based performance evaluations are currently available to assess realistic gains and limitations. We study the achievable performance of inter-cell interference-limited networks with single-user detection for slow and fast fading channels that are non-stationary but doubly underspread. The evaluation is based on channel measurements at 2.53 GHz in an urban macrocell scenario. Different beamforming techniques, a non-cooperative and two cooperative ones, are used and their robustness to the non-stationarity of the channel is evaluated. The cooperative techniques are found to be much less robust to the non-stationarity of the channel. Furthermore, an approximation of the mutual information is evaluated; it is more accurate for the cooperative techniques.

1 Introduction

The performance of next generation cellular networks is limited by inter-cell interference (ICI) due to a frequency reuse factor of 1 [1]. This is especially a concern for users at the cell-edge, which suffer from high ICI. A promising approach to mitigate ICI while maintaining fairness among all users in the network is the use of transmit beamforming with multiple antennas at the base stations (BSs) [2]. The mutual information (MI) over multiple-input and multiple-output (MIMO) channels with ICI and single-user detection was studied in, e.g., [3] and [4]. In [3], optimum signaling in terms of the sum ergodic capacity for Rayleigh frequency-flat fading channels were investigated. The optimum ergodic capacity per user for Rician frequency-flat fading channels with Kronecker structure was studied in [4]. There are only few measurement-based evaluations of the MI or related quantities in ICI-limited networks. In [5] and [6], the performance in a network with ICI and single-user detection based on measurements in urban macrocell scenarios was evaluated. More specifically, in [5], the MI of frequency-flat fading MIMO channels in a slow fading scenario was investigated at 2.53 GHz for equal power allocation across the antennas of the transmitter (TX), and, in [6], weighted sum rates of coordinated multicell techniques for frequency-flat fading MISO channels were studied at 1.8 GHz. Besides ICI, another important issue is the non-stationary of the wireless channel, e.g., the statistics of the channel process change over time [7], [8]. Beamforming techniques relying on statistical knowledge of the channel need to update this knowledge to avoid a severe performance degradation due to mismatched beamforming. The effects of mismatched beamforming based on statistical knowledge of the channel were evaluated for an urban macrocell scenario at 2.53 GHz in [8]. The analysis was, however, based on the signal-to-interference-plus-noise ratio (SINR) of users served by multiple BSs with interference from outside the network. In current system-level simulation environments, the effects of ICI and channel non-stationarity are only unsatisfactoraly modeled [9, 10]; therefore, measurement-based studies are needed to assess the performance of realistic ICI-limited cellular networks.

Contributions: We study downlink beamforming techniques with and without cooperation in an ICI-limited cellular network over time-selective, frequency-flat fading channels. The characterization is based on non-stationary, doubly underspread (DU) channels [11] measured in an urban macrocell scenario that is relevant to 3GPP Long Term Evolution (LTE). Our main contributions are as follows.

- We evaluate the achievable performance of a non-cooperative and two cooperative beamforming techniques in slow and fast fading channels in terms of the MI. An approximate evaluation of the MI is found to be more accurate for the cooperative techniques.
- We assess the robustness of the beamforming techniques to the non-stationarity of the channel by studying local quasi-stationarity (LQS) regions. We find that the cooperative techniques suffer from a high sensitivity to the non-stationarity of the channel.

Notation: $\mathbf{A}^*$, $\mathbf{A}^T$, and $\mathbf{A}^H$ denote the (element-wise) complex conjugate, the transpose, and the conjugate transpose of the matrix $\mathbf{A}$, respectively. $||\mathbf{A}||_F$ denotes the Frobenius norm of the matrix $\mathbf{A}$. $|\mathcal{A}|$ denotes the cardinality of the set $\mathcal{A}$. The expectation of a random value x is denoted by $\mathrm{E}\left\{x\right\}$. All logarithms are to the base 2.

2 System Model

We consider a downlink cellular network with N_{BS} BSs where one BS transmits to a single mobile terminal (MT) per time slot by means of beamforming. The ICI results from $N_{\text{BS}}-1$ BSs transmitting each to a single and different MT per time slot. Transmission takes place over a time-varying, frequency-flat fading multiple-input and single-output (MISO) channel with N_{TX} antennas at each BS. In the complex baseband, the matched-filtered and symbol-sampled received signal at the MT associated to BS k is

$$y_k[m] = \mathbf{L}_{\text{H},k,k}[m]\mathbf{w}_k[m]x_k[m] + \sum_{i=1,i\neq k}^{N_{\text{BS}}} \mathbf{L}_{\text{H},k,i}[m]\mathbf{w}_i[m]x_i[m] + n_k[m] \tag{1}$$

for the time slot $m \in \mathbb{Z}$. The samples $\mathbf{L}_{\text{H},k,i}[m]$ are jointly proper row vectors containing the fading values of the MISO channel from BS i to the MT associated to BS k. We assume the receiver to have channel state information (CSI), i.e., the MT associated to BS k knows the current channel realizations $\mathcal{L}_{\text{H},k}[m] = \{\mathbf{L}_{\text{H},k,i}[m] \mid i = 1, ..., N_{\text{BS}}\}$. The beamforming weights of BS i are denoted by $\mathbf{w}_i[m]$. The samples $x_i[m]$ are white jointly proper Gaussian signals transmitted from BS i with power $\text{E}\left\{|x_i[m]|^2\right\} = \sigma^2_{x,i}$; the signals of the different BSs are mutually independent. The samples $n_k[m]$ denote white jointly proper Gaussian noise with power σ^2_n. We define the instantaneous signal-to-noise ratio (SNR) for the MT associated to BS k as $\rho_k[m] = \rho_{k,k}[m]$ where $\rho_{k,i}[m]$ is the instantaneous interference-to-noise ratio (INR) for the MT associated to BS k due to ICI from BS i:

$$\rho_{k,i}[m] = \frac{||\mathbf{L}_{\text{H},k,i}[m]||^2_F}{N_{\text{TX}}}\frac{\sigma^2_{x,i}}{\sigma^2_n}. \tag{2}$$

Note that these definitions of the SNR and the INR do not contain beamforming gains.

3 Mutual Information With Beamforming

With respect to the system model in Section 2, the MI between the input and the output combined with CSI at the receiver is given by [3]

$$I_{\text{BF}}\left(x_k[m]; y_k[m], \mathcal{L}_{\text{H},k}[m]\right) = \text{E}\left\{\log\left(1 + \mathbf{L}_{\text{H},k}[m]\mathbf{Q}_k[m]\mathbf{L}^H_{\text{H},k}[m]z_k^{-1}[m]\right)\right\} \tag{3}$$

with $\mathbf{L}_{\text{H},k}[m] = \mathbf{L}_{\text{H},k,k}[m]$, the input covariance matrix of BS i denoted by $\mathbf{Q}_i[m] = \mathbf{w}_i[m]\mathbf{w}^H_i[m]\sigma^2_{x,i}$, and $z_k[m] = \sum_{i=1,i\neq k}^{N_{\text{BS}}} \mathbf{L}_{\text{H},k,i}[m]\mathbf{Q}_i[m]\mathbf{L}^H_{\text{H},k,i}[m] + \sigma^2_n$. In [12], we commented on the interpretation of (3) as performance measure for slow and fast fading channels. The MI (3) can also be approximated by [4, 12, 13]

$$I_{\text{BF}}\left(x_k[m]; y_k[m], \mathcal{L}_{\text{H},k}[m]\right) \approx \log\left(1 + \text{SINR}_k[m]\right) \tag{4}$$

where we define the SINR of the MT associated to BS k as

$$\text{SINR}_k[m] = \frac{\mathbf{w}^H_k[m]\mathbf{R}^*_k[m]\mathbf{w}_k[m]\sigma^2_{x,k}}{\sum_{i=1,i\neq k}^{N_{\text{BS}}} \mathbf{w}^H_i[m]\mathbf{R}^*_{k,i}[m]\mathbf{w}_i[m]\sigma^2_{x,i} + \sigma^2_n}. \tag{5}$$

The correlation matrix of the channel from BS i to the MT associated to BS k is $\mathbf{R}_{k,i}[m] = \text{E}\{\mathbf{L}^T_{\text{H},k,i}[m]\mathbf{L}^*_{\text{H},k,i}[m]\}$ and $\mathbf{R}_k[m] = \mathbf{R}_{k,k}[m]$. In [12], we analyzed the error of the approximation (4) by a multivariate Taylor expansion.

4 Non-Cooperative Beamforming

The first beamforming technique is non-cooperative: BS k performs beamforming to the MT associated to it based on statistical knowledge of the corresponding link only, i.e., $\mathbf{w}_k[m] = \mathbf{u}^*_{\text{max},k}[m]$ with $||\mathbf{w}_i[m]||^2_F = 1$ where $\mathbf{u}_{\text{max},k}[m]$ is the dominant eigenvector of $\mathbf{R}_k[m]$. Assuming only knowledge of $\mathbf{R}_k[m]$ at BS k, with this technique, each BS maximizes the signal power at its MT while disregarding any interference to MTs associated to other BSs.

5 SINR-Balanced Beamforming

The cooperative beamforming techniques assume that the BSs can cooperate in the sense that each BS knows the correlation matrix of the links between each BS and track. The goal of these techniques is to maximize the minimum SINR, i.e., to balance the SINRs of all MTs, while fulfilling a certain TX power constraint:

$$\begin{aligned} &\max_{\{\mathbf{w}_i | i=1,...,N_{\text{BS}}\}} \min_k \ \text{SINR}_k[m] \\ &\qquad \text{subject to} \quad \text{a) } ||\mathbf{w}_i[m]||^2_F \leq 1,\ i = 1, ..., N_{\text{BS}} \quad \text{or} \quad \text{b) } \sum_{i=1}^{N_{\text{BS}}} ||\mathbf{w}_i[m]||^2_F \leq N_{\text{BS}}. \end{aligned} \tag{6}$$

We thus impose a per-BS power constraint in case a) and a sum power constraint in case b). We attempt to solve this problem with an iterative feasibilty check using a bisection method [14]. With (4), we conclude that a balanced SINR results in an approximately balanced MI. Thus, this technique approximately balances the MI of all MTs.

6 Local Quasi-Stationarity

In the context of non-stationary channels, we want to quantify the effect of mismatched beamforming on the MI, i.e., the robustness of a beamforming technique to the non-stationarity of the channel. Mismatch refers to the use of statistical knowledge from a different time instant $m' \neq m$ for the beamforming design. Based on (3), we define the time-varying MI with mismatched beamforming as

$$I_{\text{BF}}[m,m'] = \mathrm{E}\left\{\log\left(1+\mathbf{L}_{\text{H},k}[m]\mathbf{Q}_k[m']\mathbf{L}^H_{\text{H},k}[m]z_k^{-1}[m]\right)\right\} \tag{7}$$

where the statistical mismatch is characterized by $\mathbf{Q}_k[m']$. The approximate evaluation based on (4) follows accordingly. We can use the degradation of the MI

$$\eta_I[m,m'] = 1 - \frac{I_{\text{BF}}[m,m']}{I_{\text{BF}}[m,m]} \tag{8}$$

due to the use of mismatched beamforming to obtain LQS regions [7]. Defining the threshold $\eta_{\text{th},I}$ and the set $\mathcal{M}_I[m] = \{m' \mid \eta_I[m,m'] < \eta_{\text{th},I}\}$, we obtain time-dependent LQS times

$$T_{\text{LQS}}[m] = |\mathcal{C}_I[m]|\, T_s \tag{9}$$

where $\mathcal{C}_I[m]$ is the connected subset of $\mathcal{M}_I[m]$ with maximum cardinality and containing m.

7 Channel Measurements and Data Processing

The 3GPP LTE-relevant channel measurements were performed at 2.53 GHz in 2 bands of 45 MHz [7]. The measurement campaign sequentially covers 3 BS positions with, amongst others, 25 m height in a scenario of urban macrocell type. We use a 20 MHz band between 2.49 GHz and 2.51 GHz for the estimation of the statistics. At the BS, we choose a uniform linear array with the central 4 vertically polarized elements. At the MT (passenger car), we extract 4 of the 12 vertically polarized elements of the lower uniform circular array to construct a MISO channel for each element. The elements correspond to the front (direction of motion), the back, and the two sides of the MT. All elements are made of directional patch antennas. For further details, see [7]. We preprocess the data by estimating a noise level in the time-delay domain and not considering any values below it. In order to estimate the statistical quantities, i.e., the MIs (3) and (7) and the correlation matrices, we approximate the ensemble averaging by an averaging operation over time and frequency. However, before doing so, we verify the DU assumption $\Delta\tau_{\text{max}}\Delta\nu_{\text{max}} \ll \tau_{\text{max}}\nu_{\text{max}} \ll 1$ to guarantee that the channel statistics can be estimated by averaging over time and frequency [11]. In [7], we showed that in our scenario the DU assumption is fulfilled with $\Delta\tau_{\text{max}}\Delta\nu_{\text{max}} \approx 1.16 \cdot 10^{-7}$ and $\tau_{\text{max}}\nu_{\text{max}} \approx 1.17 \cdot 10^{-4}$. Furthermore, we showed that an averaging in time over $N_t = 16$ and in frequency over $N_f = 128$ samples is a sensible choice to estimate the channel statistics. We thus average over a total of 2048 ($\approx$ 500 non-coherent) realizations [7]. We effectively remove the path loss and the shadow fading by normalizing the channel inside the time-frequency regions used to estimate the statistical quantities. This allows us to control the amount of interference. We designate by $\overline{\text{SNR}}$ and $\overline{\text{INR}}$, the mean of the instantaneous SNR and INR (2) averaged over the N_f frequency samples and the interval length N_t.

8 Results

In the following analysis, each BS has one MT track inside its cell; we associate track 10b-9a to BS 1, track 9a-9b to BS 2, and track 41a-42 to BS3, see [7]. Note that we consider a cell-edge scenario, since the 3 MTs are located inbetween the 3 BSs. To this end, we also set $\overline{\text{SNR}} = 10$ dB and $\overline{\text{INR}} = 10$ dB. We analyze the MI for different orientations of the MTs, where we choose an equal orientation for all MTs. The MTs move from the beginning towards the end of their track. The MI is given in bit/channel use (bit/c.u.), where a channel use corresponds to a time slot m in (1). In Fig. 1 a), we show the cumulative distribution function (CDF) of the MI for the different beamforming techniques. In the considered cell-edge scenario, the use of BS cooperation drastically increases the mean of the MI and it lowers the variance. We observe only a slight improvement when replacing the per BS power constraint a) with the sum power constraint b); however, note that we normalize the channel, which effectively removes the path loss and the shadowing. In order to study the robustness of the beamforming techniques against the non-stationarity of the channel, we use the definition of LQS from Section 6. We choose a threshold of $\eta_{\text{th},I} = 0.1$, i.e., we define a 10% degradation of the MI due to mismatched statistical knowledge to be the minimal loss that is not acceptable anymore. The acceptable loss defines the size of the LQS regions; a high LQS distance is equivalent to a high robustness to the non-stationarity of the channel. We observe in Fig. 1 b) that the non-cooperative beamforming technique is much more stable than the cooperative ones; this is explained by the fact that the beamforming weights only rely on the statistical channel properties of one link. Interestingly, for the cooperative techniques, the approximate (ap) evaluation of the MI, i.e., (4), leads to more accurate results than for the non-cooperative one. In Fig. 2, we show an example of the evolution of the 3 MT MIs for the cooperative beamforming technique a) and the front orientation. One can clearly see that the MIs follow the same trends due to the SINR balancing.

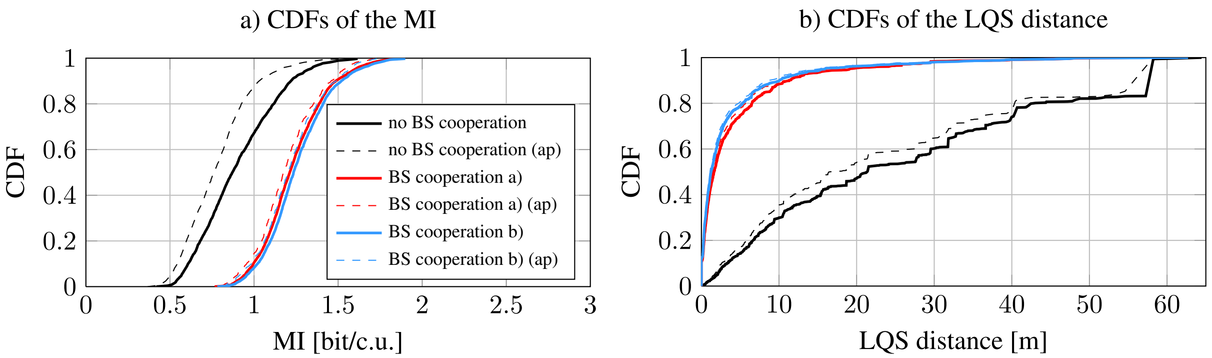

Figure 1: CDFs of the MI and the LQS distance for $\overline{\mathrm{SNR}} = 10$ dB and $\overline{\mathrm{INR}} = 10$ dB using the different beamforming techniques.

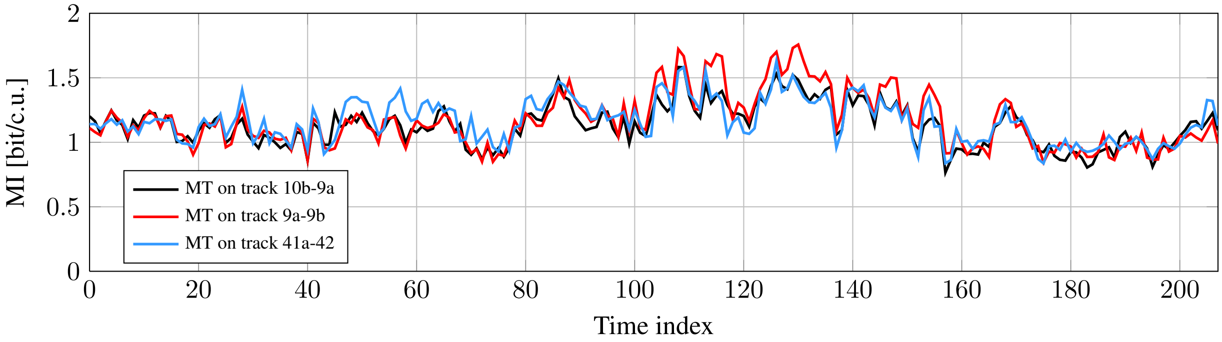

Figure 2: MIs with $\overline{\mathrm{SNR}} = 10$ dB, $\overline{\mathrm{INR}} = 10$ dB for the cooperative beamforming technique a) and the front orientation.

9 Conclusion

In this paper, we have studied the achievable performance of inter-cell interference-limited networks with single-user detection and different beamforming techniques for slow and fast fading channels. The evaluation is based on non-stationary but doubly underspread channels in an urban macrocell scenario at 2.53 GHz. The beamforming techniques, a non-cooperative and two cooperative ones, represent different extents of channel knowledge at the BSs. An approximate evaluation of the MI is found to be more accurate for the cooperative techniques. Furthermore, we evaluated the robustness of the beamforming techniques to the non-stationarity of the channel by means of LQS regions; we found that the cooperative techniques are far more sensitive to the non-stationarity of the channel.

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