IMPROVING MIMO PHASE NOISE ESTIMATION BY EXPLOITING SPATIAL CORRELATIONS

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ABSTRACT

Phase locked loops (PLL) for RF carrier synthesis often employ oscillators that insert a considerable amount of time varying phase noise into the received signal. That noise must then be removed in digital basebandreceiver. This phase noise is an indivisible superposition of noise components from receiver and transmitter. Regarding to systems with multiple transmit and receive antennas (MIMO) and if multiple PLLs for carrier synthesis are used each of the superposed phase noise processes per transmit and receive antenna pair can be measured at the receiver. This paper provides a new scheme for high SNR scenarios that exploits spatial correlation between these overlaying phase noise processes at the receiver in order to improve estimation and compensation of the phase noise. Therefore the Wiener filter approach is applied.

1. INTRODUCTION

Modern communication systems suffer – and will probably always do – from impairments due to non-ideal RF front ends. Because of strict financial constraints, RF components with non-ideal properties e.g. with nonlinearities and noise effects are used. Regarding to carrier frequency synthesis RF oscillators used in phase locked loops (PLL) often introduce a considerable amount of time varying phase alterations into the signal. This carrier signal is used for both, converting the baseband signal in the transmitter to bandpass range and, after transmission over the channel, in the receiver back to baseband. The inherent phase noise in the carrier becomes a dominant source of signal distortion in several current broadband systems like e.g. in wireline and satellite communications or directed microwave radio links.

The phase noise can be described by a time-varying random walk process with a limited bandwidth. Hence, by exploiting noise correlation in time this random process can be estimated and compensated in the receiver. Good results have been obtained employing data aided (DA) estimation algorithms [1] that extract information on the phase variations from pilot signals inserted in the data stream. A linear minimum mean square error approach (LMMSE) of an estimator for a broadband single carrier system using Wiener filters is proposed in [2]. Most other literature is concerned with the significantly different case of OFDM systems [4],[3].

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When considering the phase noise problem in the MIMO environment with N_T transmit and N_R receive antennas with each antenna having independent circuitry for carrier synthesis the task is extended to an estimation and compensation of $N_T \cdot N_R$ phase noise processes. Although the original phase noise processes are spatially uncorrelated due to the individual circuitry, it will be shown in this paper that the resulting processes as seen from the receiver are correlated in time and space. The easiest but suboptimum solution is based on the SISO approach [2] and neglects all spatial correlations using the proposed Wiener filter for each noise process. The refinement exploits the spatial noise correlation in the filter design process. As the resulting estimates of the noise processes are based on the received pilot data of all observations a *vector estimator* as described e.g. in [7] and worked out for two antennas in [5] is obtained. The reduced estimation error can be used for increasing the pilot spacing while maintaining the same overall estimation error and respectively reducing the overall number of pilots. The higher losses due to aliasing can be traded off with an increased power efficiency of the transmission. Alternatively even cheaper analogue equipment that introduces more phase noise in the system can be employed.

This remaining text is organized as follows: in Part 2 the underlying signal model and the correlations are explained. Part 3 deals with the derivation of the estimation filter from Wiener filter theory while in Part 4 the resulting estimation errors of the new estimation scheme are discussed and compared with the standard temporal estimator. The paper is concluded and summarized in Part 5.

2. PHASE NOISE IN MIMO SYSTEMS

The underlying system consists of a transmitter with N_T antennas and a receiver with N_R receive antennas. At time instance k a transmit signal vector $\boldsymbol{s}_k$ of dimension N_T is conveyed via a complex valued $N_R \times N_T$ channel matrix $\boldsymbol{H}_k$ to the receiver which results in the received signal vector

$$\boldsymbol{r}_k = \boldsymbol{J}_{Rx;k}\boldsymbol{H}_k\boldsymbol{J}_{Tx;k}\boldsymbol{s}_k + \boldsymbol{n}_k \tag{1}$$

of dimension N_R. The diagonal matrices $\boldsymbol{J}_T$ and $\boldsymbol{J}_R$ with dimensions N_T and N_R represent the transmitter respectively receiver phase noise effects. Their diagonal elements consist of a complex phasor of magnitude 1 with the phase $\phi^{(T)}_{n;k}$ for the transmitter and $\phi^{(R)}_{m;k}$ for the receiver phase noise. The noise vector $\boldsymbol{n}_k$ is composed of N_R uncorrelated white gaussian noise samples with the auto-covariance matrix $\boldsymbol{C_{nn}} = \sigma_n^2 \boldsymbol{I}_{N_R}$ wherein $\boldsymbol{I}_{N_R}$ is the unity matrix of dimension $N_R \times N_R$. Strictly speaking the AWGN is also rotated by the receiver phase noise but AWGN is invariant to this phase rotation.

Then, the signal at the m-th receive antenna can be expressed as

$$r_{m;k} = \sum_{n=1}^{N_T} e^{j[\phi_{n;k}^{(T)}+\phi_{m;k}^{(R)}]} h_{mn;k} s_{n;k} + n_{m;k} \quad (2)$$

i.e. each of the N_T interfering transmitted signals is distorted by the sum-process

$$\varphi_{mn;k} = \phi_{n;k}^{(T)} + \phi_{m;k}^{(R)}. \quad (3)$$

In principle the $N_T \cdot N_R$ sum-processes can be estimated using N_T orthogonal pilot symbols. But the conclusion back on their original processes is not possible as the next paragraph will show. If we gather these original phase noise processes in one vector

$$\boldsymbol{\phi}_k = \left(\phi_{1;k}^{(T)}, \phi_{2;k}^{(T)}, \dots \phi_{N_T;k}^{(T)}, \phi_{1;k}^{(R)}, \dots \phi_{N_R;k}^{(R)}\right)^T \quad (4)$$

and the sum-processes in another vector

$$\boldsymbol{\varphi}_k = (\varphi_{11;k}, \dots \varphi_{1N_R;k}, \varphi_{21;k}, \dots \varphi_{2N_R;k}, \dots \varphi_{N_T N_R;k})^T \quad (5)$$

a matrix $\boldsymbol{G}$ interrelates the sum-processes to their original counterparts:

$$\boldsymbol{\varphi}_k = \boldsymbol{G}\boldsymbol{\phi}_k. \quad (6)$$

The resulting $(N_R N_T) \times (N_R + N_T)$ matrix

$$\begin{aligned} \boldsymbol{G} &= \left(\boldsymbol{I}_{N_T} \otimes \mathbf{1}_{N_R} \quad \mathbf{1}_{N_T} \otimes \boldsymbol{I}_{N_R}\right) \\ &= \left(\boldsymbol{g}_1 \, \boldsymbol{g}_2 \, \dots \, \boldsymbol{g}_{N_T+N_R}\right) \end{aligned} \quad (7)$$

that can be divided into its column vectors $\boldsymbol{g}_i$ has not full column rank due to the systematic nature of the matrix. Therein $\mathbf{1}_L$ denotes a column vector of length L with one entries and $\otimes$ describes the Kronecker product of two matrices. The first column vector $\boldsymbol{g}_1$ can always be expressed as the sum of the last N_R column vectors minus the sum of the rest of the first $N_T - 1$ column vectors:

$$\boldsymbol{g}_1 = \sum_{v=N_T+1}^{N_T+N_R} \boldsymbol{g}_v - \sum_{v=2}^{N_T} \boldsymbol{g}_v \quad (8)$$

Hence it is not possible to extract the processes $\phi_{n;k}^{(T/R)}$ from the sums: the pilot data based approach on estimating the phase noise does not allow to conclude on the original processes $\phi_{n;k}^{(T)}$ and $\phi_{m;k}^{(R)}$ but on their sum-process $\varphi_{mn;k}$. A plausible non-mathematical explanation for that is a missing of a reference phase.

The phase noise itself is usually modelled as time continuous random walk process i.e. the process is given by the integration

$$\phi(t) = \int_{-\infty}^{t} \xi(\tau) d\tau \quad (9)$$

wherein $\xi(\tau)$ can be assumed zero mean Gaussian. Therefore the phase noise process $\phi(t)$ is completely described by its autocorrelation function

$$r_\phi = \int_{-\infty}^{\infty} \phi(t)\phi(t+\tau) dt \quad (10)$$

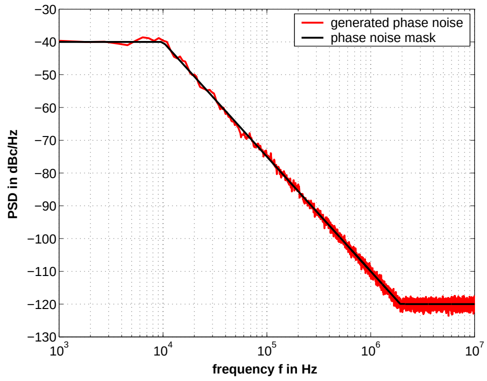

Fig. 1. Typical example PSD of a PLL

(here as temporal mean thanks to ergodicity of ϕ) or equivalently by its power spectral density (PSD). Thus, these phase noise processes are characterized by their PSD given in so called phase noise masks e.g. like depicted for a devised but representative characteristic in Fig. 1.

Although the original processes cannot be estimated their occurrence in the different sum-processes can be exploited in terms of their correlations. Presuming uncorrelated processes at receiver and transmitter, that means for a common transmitter n and receive antennas m_1 and m_2:

$$\begin{aligned} E\{\varphi_{m_1 n}\varphi_{m_2 n}\} &= E\left\{(\phi_{n;k}^{(T)}+\phi_{m_1;k}^{(R)})(\phi_{n;k}^{(T)}+\phi_{m_2;k}^{(R)})\right\} \\ &= E\left\{\phi_{n;k}^{(T)2}\right\} + E\left\{\phi_{m_1}^{(R)}\phi_{m_2;k}^{(R)}\right\} \end{aligned} \quad (11)$$

with $n \in \{1 \dots N_T\}$ and $m_{1,2} \in \{1 \dots N_R\}$. For independent phase noise processes i.e. independent PLLs at the receiver the second sum term vanishes.

$$E\{\varphi_{m_1 n}\varphi_{m_2 n}\} = E\left\{\phi_n^{(T)2}\right\} \quad (12)$$

A similar relation holds for signals from a common receive antenna

$$E\{\varphi_{mn_1}\varphi_{mn_2}\} = E\left\{\phi_m^{(R)2}\right\}. \quad (13)$$

Although the phase noise signal as such is not band-limited, in this paper, aliasing effects will not be taken into account in the derivation due to the limitations in space but the results also comprise degradation due to insufficient sampling frequency.

3. WIENER FILTER DESIGN

The Wiener filter approach applied to our example is based on a set of observations $\eta_{mn;k}$ of the phases $\varphi_{mn,k}$ in radians from which an estimate $\widehat{\varphi}_{mn,k}$ is computed. For this application in the high SNR region, an observation consists of the phase noise sum-process and additive noise that can be approximated in the high SNR range as real valued white Gaussian noise $\tilde{n}_{mn;k}$ with power $\sigma_n^2/2$ (compare eq. (2)):

$$\eta_{mn;k} = \arg\left(r_{m;k} s_{n;k}^*\right) \approx \varphi_{mn;k} + \tilde{n}_{mn;k}. \quad (14)$$

F temporal observations of one process at pilot rate $\frac{1}{P_S}$ are then gathered in a column vector $\boldsymbol{\eta}_{mn}$. With the short cuts $L = \lfloor F/2 \rfloor \cdot P_S$ and $\lambda = l \cdot P_S$ this vector becomes

$$\boldsymbol{\eta}_{mn;\lambda} = (\eta_{mn;\lambda-L} \, \eta_{mn;\lambda-L+P_S} \cdots \eta_{mn;\lambda-L+FP_S})^T \quad (15)$$

In order to gain the vector $\boldsymbol{\eta}_\lambda$ of all observations these vectors $\boldsymbol{\eta}_{mn;\lambda}$ are stacked in order of columns before rows such that

$$\boldsymbol{\eta}_\lambda = (\boldsymbol{\eta}_{11;\lambda}^T \cdots \boldsymbol{\eta}_{N_R 1;\lambda}^T \, \boldsymbol{\eta}_{12;\lambda}^T \cdots \boldsymbol{\eta}_{N_R N_T;\lambda}^T)^T \tag{16}$$

From the auto-covariance matrix of these observations

$$\boldsymbol{C}_{\boldsymbol{\eta\eta}} = \boldsymbol{C}_{\boldsymbol{\varphi\varphi}} + \boldsymbol{C}_{\tilde{\boldsymbol{n}}\tilde{\boldsymbol{n}}} \tag{17}$$

and the cross-covariances $\boldsymbol{C}_{\boldsymbol{\eta\varphi}}$ between observations and the estimated parameter, it is straight forward to design the respective Wiener filters corresponding to a symmetric, temporal observation window of length K. The Wiener-Hopf equation is written e.g. according to [7] as

$$\boldsymbol{w} = \boldsymbol{C}_{\boldsymbol{\eta\eta}}^{-1} \boldsymbol{C}_{\boldsymbol{\eta\varphi}}. \tag{18}$$

The aim of the following paragraph is to compute the respective covariance matrices. The auto-covariance matrix from (17) is found to have block-matrix structure :

$$\boldsymbol{C}_{\boldsymbol{\eta\eta}} = \begin{pmatrix} \boldsymbol{C}_{\eta_{11}\eta_{11}} & \boldsymbol{C}_{\eta_{11}\eta_{21}} & \cdots & \boldsymbol{C}_{\eta_{11}\eta_{N_R N_T}} \\ \boldsymbol{C}_{\eta_{21}\eta_{11}} & \boldsymbol{C}_{\eta_{21}\eta_{21}} & \cdots & \boldsymbol{C}_{\eta_{21}\eta_{N_R N_T}} \\ \vdots & \vdots & & \vdots \\ \boldsymbol{C}_{\eta_{N_R 1}\eta_{11}} & \boldsymbol{C}_{\eta_{N_T 1}\eta_{21}} & \cdots & \boldsymbol{C}_{\eta_{N_T 1}\eta_{N_R N_T}} \\ \boldsymbol{C}_{\eta_{12}\eta_{11}} & \boldsymbol{C}_{\eta_{12}\eta_{21}} & \cdots & \boldsymbol{C}_{\eta_{12}\eta_{N_R N_T}} \\ \vdots & \vdots & & \vdots \\ \boldsymbol{C}_{\eta_{N_R N_T}\eta_{11}} & \boldsymbol{C}_{\eta_{N_R N_T}\eta_{21}} & \cdots & \boldsymbol{C}_{\eta_{N_R N_T}\eta_{N_R N_T}} \end{pmatrix}. \tag{19}$$

For the partial auto-covariance matrices $\boldsymbol{C}_{\eta_{\alpha\beta}\eta_{\gamma\delta}}$ where $\alpha, \gamma \in [1, N_R]$ and $\beta, \delta \in [1, N_T]$ four cases are distinguished:

(1.) $\alpha = \gamma$ and $\beta = \delta$ (like for the single antenna phase estimator)

$$\mathbf{C}_{\eta_{\alpha\beta}\eta_{\alpha\beta}} = \mathbf{C}_{\varphi_{\alpha\beta}\varphi_{\alpha\beta}} + \frac{\sigma_n^2}{2} \cdot \mathbf{I}. \tag{20}$$

(2.) $\alpha \neq \gamma$ and $\beta = \delta$ (observation signal streams incorporating the same transmit oscillator)

$$\mathbf{C}_{\eta_{\alpha\beta}\eta_{\gamma\beta}} = \mathbf{C}_{\varphi_{\alpha\beta}\varphi_{\gamma\beta}}. \tag{21}$$

(3.) $\alpha = \gamma$ and $\beta \neq \delta$ (Observation signal streams incorporating the same receive oscillator)

$$\mathbf{C}_{\eta_{\alpha\beta}\eta_{\alpha\delta}} = \mathbf{C}_{\varphi_{\alpha\beta}\varphi_{\alpha\delta}}. \tag{22}$$

(4.) $\alpha \neq \gamma$ and $\beta \neq \delta$ (Observation signal streams are totally independent)

$$\mathbf{C}_{\eta_{\alpha\beta}\eta_{\gamma\delta}} = \mathbf{C}_{\varphi_{\alpha\beta}\varphi_{\gamma\delta}}. \tag{23}$$

In the cases $2, 3$ and 4 the AWGN term disappears as the respective noise processes are spatially uncorrelated. Strictly speaking spatial independency only holds for pilot schemes where, at one time instance, a signal is transmitted from one antenna and zeros from the others. For other employed orthogonal pilot schemes this approach still serves as a good approximation. The respective covariance matrices are then composed of the covariances

$$r(\lambda) = r_{\alpha\gamma}^{(R)}(\lambda) + r_{\beta\delta}^{(T)}(\lambda) \tag{24}$$

wherein $\lambda = (k_0 - k_1) \cdot P_S$ describes the time offset between two observations and $r_{ab}^{(T/R)}$ mirrors the partial correlation from transmitter or receiver side. For independent PLLs at the receiver the first summand and for independent transmit circuits the second one disappears. Hence also the special case of one common carrier synthesis for all antennas on one side is part of this derivation if the covariances $r(\lambda)$ adopt the same value of full correlation. According to [2] these covariances can be calculated from

$$r_{ab}^{(R/T)}(\lambda) = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{ab}(\omega) e^{j\omega\lambda} d\omega \tag{25}$$

The a and b herein represent some α, β respectively γ, δ from above and S_{ab} represents the common spectrum of the respective noise processes.

The cross-covariances for the interpolated estimate at time instance l for the sum-process of transmitter γ and receiver δ in

$$\boldsymbol{C}_{\boldsymbol{\eta}\boldsymbol{\varphi}_{\gamma\delta;l}} = \Big(\boldsymbol{C}_{\boldsymbol{\eta}_{11}\boldsymbol{\varphi}_{\gamma\delta;l}}^T \boldsymbol{C}_{\boldsymbol{\eta}_{21}\boldsymbol{\varphi}_{\gamma\delta;l}}^T \cdots \boldsymbol{C}_{\boldsymbol{\eta}_{N_R 1}\boldsymbol{\varphi}_{\gamma\delta;l}}^T \boldsymbol{C}_{\boldsymbol{\eta}_{12}\boldsymbol{\varphi}_{\gamma\delta;l}}^T \cdots \\ \boldsymbol{C}_{\boldsymbol{\eta}_{N_R 2}\boldsymbol{\varphi}_{\gamma\delta;l}}^T \cdots \boldsymbol{C}_{\boldsymbol{\eta}_{N_R N_T}\boldsymbol{\varphi}_{\gamma\delta;l}}^T \Big)^T \tag{26}$$

can be similarly determined. The entries of the submatrices arise to

$$p_l(\lambda) = p_{\alpha\gamma;l}^{(R)}(\lambda) + p_{\beta\delta;l}^{(T)}(\lambda). \tag{27}$$

The partial terms are also obtained by integrating the respective overlapping components of their spectra with a and b taken as $\alpha, \beta, \gamma, \delta$ according to the above case distinctions

$$p_{ab;l}^{(R/T)}(\lambda) = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_{ab}(\omega) e^{j\omega\lambda} e^{j\omega \frac{l}{P_S}} d\omega \tag{28}$$

which is for independent circuits also zero if $a \neq m$ or respectively $b \neq n$. As described in [7] the mean square error (MSE) of a LMMSE estimate is given by

$$J_{min} = E_l\{J_{min;l}\} = E_l\{C_{\varphi\varphi} - \boldsymbol{w}_l^T \boldsymbol{C}_{\eta\varphi}\} \tag{29}$$

where E_l is understood as the expectation with respect to l. As $p_{ab;l}^{(R/T)}$ and $r_{ab}^{(R/T)}$ cannot be evaluated analytically with reasonable effort the covariance matrices have been calculated with help of a computer and, thus, the MSE evaluation is only semi-analytically.

The unknown channel phases from the channel matrix $\boldsymbol{H}$ from (1) still reside in the measured phases $\eta_{mn;k}$. This unknown phase alters the actual phase model significantly as the observations also comprise channel phases that break up the correlations:

$$\eta_{mn;k} = \varphi_{mn;k} + \arg(h_{mn;k}) + \tilde{n}_{mn;k} \tag{30}$$

But this drawback can be circumvented if initial estimates of all sum-processes e.g. from the conventional LMMSE estimators are subtracted from the phase in (14):

$$\eta'_{mn;\lambda} = \eta_{mn;\lambda} - \widehat{\varphi}_{mn;0} \tag{31}$$

and this subtracted offset $\boldsymbol{\eta}_{mn;0}$ is added again to the estimate that is computed from the above $\boldsymbol{\eta}'_{mn;\lambda}$ afterwards. Therefore the employed estimator is resumed by

$$\widehat{\varphi}_{mn;l} = \widehat{\varphi}'_{mn;l} + \widehat{\varphi}_{mn;0} = \boldsymbol{w}_l^T \cdot \boldsymbol{\eta}'_\lambda + \widehat{\varphi}_{mn;0}. \tag{32}$$

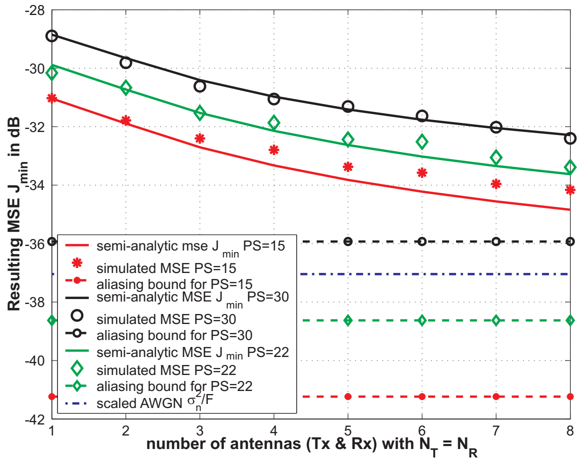

Fig. 2. MSE in dB vs. the number of antennas

4. RESULTS

This section discusses the behavior of the enhanced estimator with respect to a MIMO scenario. The results are restricted to the case of an equal number of transmit and receive antennas although the theory holds for other configurations. Phase noise according to the mask in Fig. 1 has been generated by filtering white gaussian noise in the frequency domain using the overlap save technique. The resulting PSD of the noise is also presented in the same figure. As the filters are dimensioned for the design $\mathrm{SNR} = 25\,\mathrm{dB}$, the simulations operate at the same ratio. The phase observations have been unwrapped before the estimation in order to avoid the phase ambiguity in 2π.

The required data rate is chosen to be $R_b = 155.52\frac{\mathrm{Mbit}}{\mathrm{s}}$ according to the STM-1 specification in the SDH standard. Since for reasons of easy comparison symbol rate $R_s = \frac{R_b/N_t}{k_b}$ and system bandwidth remain constant i.e. the number of bits per Symbol depends on the number of transmit antennas $k_b(N_T) = \frac{k_b(1)}{N_T}$. The pilot spacing is also not adapted. Hence the conveyable information is determined by 2 conflicting effects: increasing N_T makes, on the one hand, more orthogonal pilots necessary which decreases the number of data symbols per transmission with the factor $\frac{P_S - N_T}{P_S}$. On the other hand the number of transmittable symbols at one time instance is increased by the factor N_T. From the point of view of the phase estimation this is the best approach as bandwidth of the estimation is held constant by equal pilot spacing and equal symbol duration while from information theoretic point of view the transinformation quickly decreases when N_T gets close to the pilot spacing P_S.

Fig. 2 displays the resulting MSE in dB versus the number of antennas semi-analytically and simulated for different pilot spacings: $P_S = 15$ red line (semi-analytic) with asterisks (simulation), $P_S = 22$ in green with diamonds and $P_S = 30$ in black with circles.

The simulations for 1 antenna systems represent the reference as they exactly indicate the performance for the simple LMMSE estimator (even if more than 1 antenna is applied) neglecting the spatial correlations [2]. That means the pilot spacing for a 4×4 system can be enhanced from 15 to 30 without loss in the MSE if this introduced MIMO LMMSE estimator is used instead. Alternatively, the diagram presents that this new estimator achieves theoretically more than $3\,\mathrm{dB}$ gain for all pilot spacings and $N_T = N_R = 8$ antennas compared to the former estimator although simulations show $1\,\mathrm{dB}$ gain less for low pilot spacings.

AWGN scaled by the observation interval F is additionally displayed as dash-dotted line serving as a lower bound for low pilot spacing while for higher pilot spacing aliasing plotted with dashed lines for the different pilot spacings obviously dominates the estimation performance.

5. CONCLUSION

This paper provides an enhanced Wiener filtering scheme for the estimation of phase noise in high SNR scenarios. By exploiting the resulting spatial correlations of the phase noise processes as seen from the receiver the estimation performance in terms of the MSE of the phase estimates can be determined semi-analytically and with simulations. Both results show good conformance.

Beneath the correlations aliasing remains an important issue as with multi-antenna systems the number of orthogonal pilots is as high as the number of introduced data streams which is upper bounded by the number of transmit antennas.

The additional effort for the enhanced estimator compared to the simple SISO estimator is given by the number of observations used (optimally $N_T \times N_R$). On the one hand for each estimate of a sum-process a filter bank for each observed sum-process is used: overall $N_T^2 N_R^2$ filter banks are used but some filter outputs can be reused. But on the other hand the sets of coefficients that need to be stored remain limited: the association of a filter bank to an observed sum-process is just permuted for each sum-process to be estimated. Thus with the advance in processing speed of new hardware the additional complexity will be easy to handle.

6. REFERENCES

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