Synchronization of Large Carrier Frequency Offsets at Low Signal-to-Noise Ratios

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***Abstract*—In this paper, we examine synchronization of large carrier frequency offsets in low signal-to-noise ratio (SNR) environments. We show that a misadjustment of the receive filter in terms of its center frequency involves a significant SNR degradation that cannot be compensated for after receive filtering. In order to avoid degradations in the bit error rate of the system, we propose an iterative approach to circumvent this problem.**

I. Introduction

With the invention of Turbo Codes [1] and the rediscovery of Gallager's LDPC codes by MacKay [2], reliable transmission can be achieved at significantly reduced signal-to-noise ratios (SNR) as compared to before. However, when burst transmission is considered, synchronization is a major issue at these low SNRs.

At low SNRs, carrier frequency offsets that occur due to oscillator inaccuracies are usually combatted by data-aided (DA) synchronization algorithms [3]. These algorithms are commonly subject to a trade-off: they either provide very accurate estimates of the frequency offset, or they cover a large frequency range, meaning that they have a large estimation range. Well-known DA carrier frequency estimators are e.g. [4], [5] and [6].

All these works consider frequency offset synchronization after receive filtering. However, if a large frequency offset ($\Delta f T_s \gtrsim 0.05$) is not reduced in the analogue part of the receiver, it entails a misadjustment of the receive filter in terms of its center frequency which in turn leads to inter-symbol interference and an SNR degradation. This degradation cannot be compensated for after filtering which is intuitively understandable because signal energy once lost during the filter process cannot be recovered.

This problem was noticed in [3]. In there, they propose a bank of filters, where each filter is adjusted to a difference frequency. After the bank of filters, an energy measurement is conducted and the filter with the highest energy throughput is picked. The output signal of this filter is then further processed. Besides the rather high implementation complexity, the main drawback of this approach when used in conjunction with low SNRs is that the probability of a wrong choice of filter is not zero, resulting again in a performance degradation.

As already indicated, turbo decoding is highly sensitive to small SNR losses due to a misadjusted receive filter because its operating point is at the transition from the waterfall region to the error floor. However, even though DA frequency synchronization also exhibits a threshold behavior [7], it is rather robust against small SNR degradation as it is usually not operated near this threshold. In this paper, we propose a method to profit from this robustness, by pre-correcting the (stored) received signal with a DA frequency estimate and then re-processing it.

We examine the proposed algorithm in the context of DVB–RCS [8], which is a standard that defines an uplink satellite channel from domestic satellite terminals. The implementation of this standard is very challenging due to transmission at low SNR coupled with short burst lengths and low-cost user equipment, i.e. unstable local oscillators and therefore high frequency offsets [9].

The paper is structured as follows: Section II introduces the considered transmission model and Section III summarizes the main aspect of DVB–RCS. The approach of iterative receive filtering is explained in Section IV and simulation results are shown in Section V. Section VI concludes the paper.

II. Transmission Model

The transmission model considered in this paper is shown in Fig. 1. Information bits are grouped into packets of length L, turbo-encoded and mapped onto a modulation alphabet $\mathcal{A}$. The resulting K_d data symbols are then transmitted over an additive white Gaussian noise (AWGN) channel together with K_p pilot symbols. The total number of symbols per burst is denoted as $K = K_d + K_p$.

As the DVB-RCS standard defines rather strict constraints on symbol timing, perfect timing synchronization can be considered a realistic assumption. The received baseband signal before receive filtering and decimation is denoted as y_l, where l is the time index corresponding to $T = T_s/n$, with T_s being the symbol duration and n being the oversampling factor. y_l can be given as

$$y_l = \left(\sum_{k=0}^{K} a_k h(lT - kT_s) \right) \cdot e^{j(2\pi \Delta f T l + \vartheta)} + \tilde{n}_l, \qquad (1)$$

with a_k being the transmit symbol at sampling instant k and h being the non-causal RRC filter impulse response according to [8] with roll-off factor $\alpha = 0.35$. $\tilde{n}_l$ denotes the samples of complex-valued AWGN before filtering. The received symbol at sampling instant k after (ideal) matched filtering is given as

$$r_k = a_k \cdot e^{j(2\pi \Delta f T_s k + \vartheta)} + n_k. \qquad (2)$$

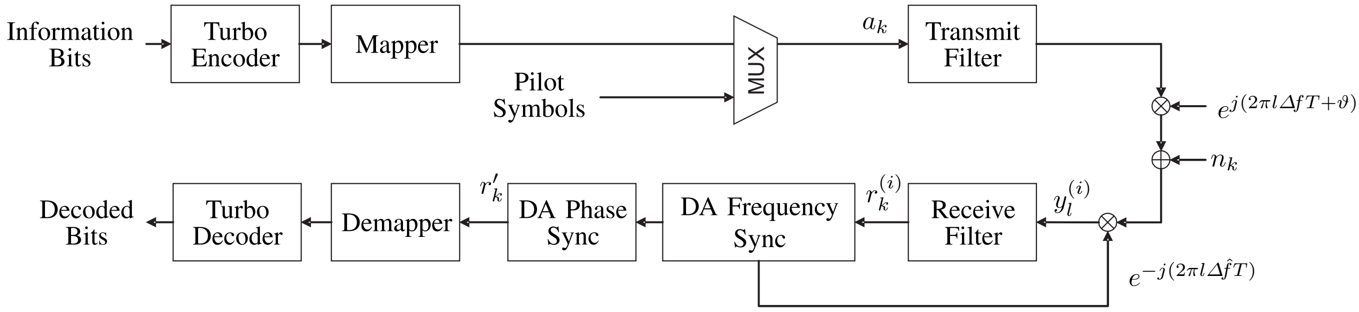


Fig. 1. System model of proposed approach

Note that in case of non-ideal receive filtering r_k contains distortions due to inter-symbol interference and suffers from an SNR loss.

Unit energy symbols are assumed, i.e. $E_s = |a_k|^2 = 1$. Furthermore, n_k are the filtered samples of complex-valued AWGN with independent real and imaginary parts, each having zero-mean and variance $N_0/(2E_s)$. In addition to the noise component, a phase offset ϑ and a frequency offset Δf are introduced by the physical channel and the local oscillators. As the DVB–RCS standard employs short bursts, phase and frequency offset can assumed to be constant for the duration of one burst.

For later use, we define the K-dimensional vector $\boldsymbol{r}$ and the $(n \cdot K)$-dimensional vector $\boldsymbol{y}$ as the concatenation of the symbols r_k and y_l, respectively.

III. DVB–RCS Parameters

DVB–RCS disposes two types of traffic bursts, the ATM burst with the length $L = 424$ and the MPEG2 burst with $L = 1504$. As the waterfall region for a turbo coded frame is usually more distinct the longer the frame is, the MPEG2 burst is more critical for our problem. Therefore, we limit our investigations on the MPEG2 burst. As DVB–RCS makes use of frequency hopping, bursts are generally neither transmitted in consecutive time-slots nor at the same frequency. Thus, receiver synchronization needs to be performed burst by burst.

Since DVB–RCS terminals are supposed to be rather low-cost, the frequency offset can be significant [9]. An offset of up to 20kHz may occur. Depending on the symbol rate, this may result in normalized offsets $\Delta f T_s$ up to 0.32, that cannot be handled by the receive filter. Therefore, one has to deal with a pre-compensation before matched filtering at the receiver.

Earlier investigations in [10] have shown that for DA frequency synchronization a pre-/mid-/postamble (P-M-P) structure with a percentage $\eta = K_p/K = 0.08$ is close to optimal for the MPEG2 burst. The P-M-P pilot constellation is dimensioned such that pre- and postamble consist of $K_p/4$ each. The residual $K_p/2$ pilot symbols are distributed equally between pre- and postamble.

IV. Frequency Synchronization Prior to Matched Filtering

As the received signal before matched filtering suffers from aliasing, data-aided frequency synchronization is usually performed after the receive filter. However, when large frequency offsets occur due to oscillator inaccuracies, the (no longer matched) receive filter cuts off a significant amount of signal energy. The SNR degradation versus the frequency offset is illustrated in Fig. 2 for different roll-off factors. The filter given in [8] uses the roll-off factor $\alpha = 0.35$. Note that the higher the roll-off factor is chosen, the less energy is lost due to a misadjusted receive filter.

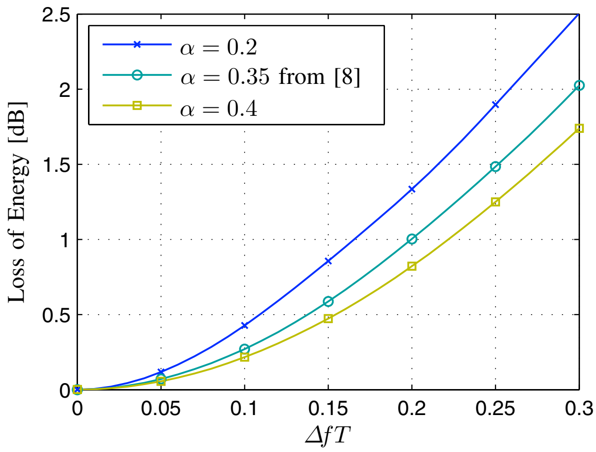


Fig. 2. SNR loss of the receive filter versus $\Delta f T_s$

In order to circumvent this SNR degradation, one common approach, which is for instance described in [3], uses a bank of N_{MF} filters as shown exemplarily in Fig. 3. The choice of the correct pre-correction $\widetilde{\Delta f}_j$, and therefore of the best filter, is usually based on an energy measurement. Hence, the signal from the output of this filter is chosen for further processing.

However, especially at low SNR the choice of the filter is not forcefully optimal. Even if a correct choice could be guaranteed, the resolution can never be as accurate as in our approach and definitely involves an SNR loss.

In our approach that is depicted in Fig. 1, we store the received signal $\boldsymbol{y}^{(0)}$ before receive filtering. We then filter it with the root raised cosine filter defined in [8] and obtain the signal $\boldsymbol{r}^{(0)}$. As already indicated in the introduction, this receive filter is no longer matched due to the (large) frequency offset of the channel. Therefore, $\boldsymbol{r}^{(0)}$ suffers from an SNR degradation as indicated in Fig. 2. However, accepting

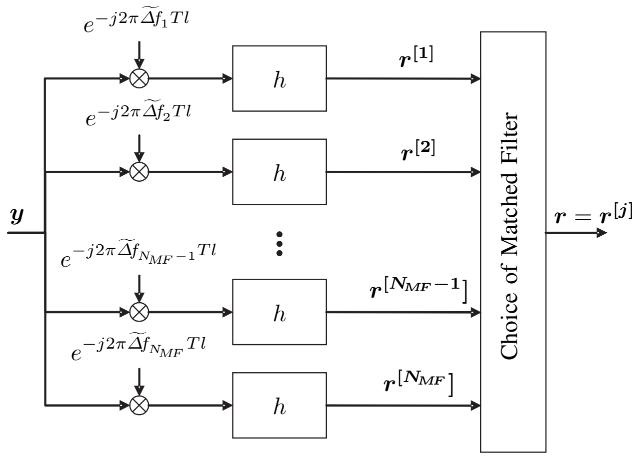


Fig. 3. Filter bank approach from [3]

this degradation and performing data-aided (DA) frequency estimation anyway, yields estimation results for the frequency offset that are hardly degraded by the SNR loss (due to the relatively weak slope of the attainable mean squared estimation error (MSEE) versus the SNR. The respective MSEEs

$$\varepsilon_{\text{MSE}}\{\Delta\hat{f}T_s\} = \mathrm{E}\Big\{(\Delta f - \Delta\hat{f})^2 T_s^2\Big\} \tag{3}$$

are shown in Tab. I.

TABLE I
MSEE FOR DA FREQUENCY ESTIMATION BASED ON $\boldsymbol{r}^{(0)}$; $E_b/N_0 = 1.2\,\text{dB}$; $r = 1/3$

	$\varepsilon_{\text{MSE}}\{\Delta\hat{f}T_s\}$
$\Delta fT_s = 0$	$1.05 \cdot 10^{-10}$
$\Delta fT_s = 0.05$	$1.07 \cdot 10^{-10}$
$\Delta fT_s = 0.1$	$1.14 \cdot 10^{-10}$
$\Delta fT_s = 0.2$	$1.42 \cdot 10^{-10}$
$\Delta fT_s = 0.3$	$1.82 \cdot 10^{-10}$

Making use of this estimate $\Delta\hat{f}T_s$, the received signal $\boldsymbol{y}^{(0)}$ is corrected, yielding:

$$y_l^{(1)} = y_l^{(0)} \cdot e^{-j(2\pi l\Delta\hat{f}T)}. \tag{4}$$

The sequence $\boldsymbol{y}^{(1)}$ consisting of the concatenation of all $y_l^{(1)}$ is then again processed by the receive filter and, subsequently synchronized in frequency and phase. Since the estimation performance obtained in the first iteration is accurate enough for the given offsets, it is sufficient to have one single iteration between the DA frequency synchronization unit and the matched filter. Therefore, the i in Fig. 1 is either zero or one.

For details concerning the utilized DA frequency estimator and DA phase estimator, please refer to [10]. It is important to note that the estimators are efficient in the considered SNR region, meaning that their mean squared estimation error (almost) coincides with the DA Cramér-Rao Bound for the considered pilot constellation.

V. Simulations

As already indicated in Section III, the simulation results in this paper are presented for the MPEG2 burst with $L = 1504$. Details concerning the code and the interleaver can be found in the standard [8]. The modulation is Gray-mapped QPSK. We consider a code rate of $r = 1/3$.

Fig. 4 shows the convergence in terms of the frame error rate (FER) versus the number of turbo decoding iterations. The curves correspond to different initial frequency offsets ΔfT_s. Please note that in Fig. 4(a) the curve with the hexagram markers, corresponding to $\Delta fT_s = 0.3$, is not missing, but is underneath the curve with the diamond markers ($\Delta fT_s = 0.2$) and that all curves coincide in Fig. 4(b).

Fig. 4(a) depicts the results that are obtained when both synchronization and detection are performed after receive filtering the sequence $\boldsymbol{y}^{(0)}$. As already indicated, these results severely suffer from the SNR degradation due to the fact that the filter is no longer matched. As a turbo code is employed for error protection, this SNR loss has fatal effects on the FER due to its operating point near the waterfall region.

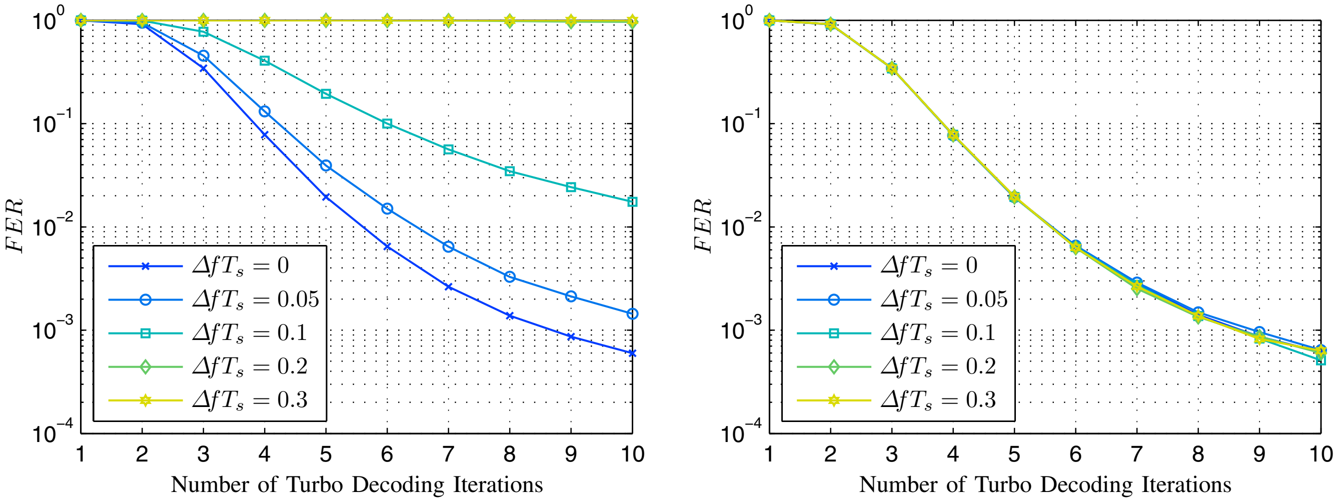
The results shown in Fig. 4(b) are based on the proposed approach depicted in Fig. 1. The signal $\boldsymbol{y}^{(0)}$ is here filtered and subsequently used for DA frequency synchronization. Afterwards, frequency pre-compensation according to (4) is applied. Then, $\boldsymbol{y}^{(1)}$ is newly filtered yielding $\boldsymbol{r}^{(1)}$. This sequence is then used for DA synchronization (frequency and phase) and afterwards for detection. It is visible, that at the cost of slightly increased complexity (receive filtering and DA frequency synchronization is performed twice) an enormous performance gain for large frequency offsets ΔfT_s can be obtained, as compared to a system where the detection is based on $\boldsymbol{r}^{(0)}$.

A FER comparison of our approach against the system shown in Fig. 3 is not carried out for two reasons. Firstly, as due to the discreteness of trial frequencies $\Delta\tilde{f}$ in Fig. 3, it is clear that its performance will be degraded as compared to our approach. Secondly, its complexity is higher (due to multiple filtering of the signal).

Note, that it is possible to further refine the frequency and phase estimate by a turbo synchronization unit, as proposed in [11]. However, investigations in [10] have shown that the performance gains in terms of the achievable FER are barely visible for the MPEG2 burst in the context of DVB–RCS. There, it is demonstrated that the accuracy of a feed-forward DA frequency and phase synchronization is sufficient.

VI. Conclusion

The main issue with large frequency offsets is that the receive filter is no longer matched and, thus, entails inter-symbol interference and a loss in the SNR. These degradations may be fatal for turbo coded systems as they are usually operated at the transition from the waterfall region to the



(a) Detection based on $\boldsymbol{r}^{(0)}$

(b) Detection based on $\boldsymbol{r}^{(1)}$

Fig. 4. FER vs. Iterations; $L = 1504$, $E_b/N_0 = 1.2\,\text{dB}$, $r = 1/3$, $\eta = 0.08$, QPSK

error floor. In this paper, we demonstrate that DA frequency estimation is relatively robust against distortions due to non-ideal receive filtering. We make use of this robustness and produce a frequency estimate that is subsequently used to pre-correct the (stored) received signal before receive filtering. Afterwards, matched filtering, synchronization and turbo decoding is performed on this pre-corrected sequence. We show by means of simulation that the performance gain achievable by this approach is high.

References

[1] C. Berrou, A. Glavieux, P. Thitimajshima, "Near Shannon Limit Error-Correcting Coding and Decoding: Turbo-Codes," in *Proc. of IEEE International Conference on Communications (ICC)*, vol. 2, Geneva, Switzerland, May 1993, pp. 1064–1070.

[2] D.J.C. MacKay, R.M. Neal, "Near Shannon Limit Performance of Low Density Parity Check Codes," *IEE Electronics Letters*, vol. 32, no. 18, pp. 1645–1646, Aug. 1996.

[3] H. Meyr, M. Moeneclaey, S. Fechtel, *Digital Communication Receivers: Synchronization, Channel Estimation and Signal Processing*, 1st ed. New York, NY: John Wiley & Sons, 1998.

[4] M. P. Fitz, "Planar Filtered Techniques for Burst Mode Carrier Synchronization," in *Proceedings of IEEE Globecom*, Phoenix, Arizona, USA, Dec. 1991.

[5] M. Luise, R. Reggiannini, "Carrier Frequency Recovery in All-Digital Modems for Burst-Mode Transmissions," *IEEE Transactions on Communications*, vol. 43, no. 2/3/4, pp. 1169–1178, Feb.-Apr. 1995.

[6] U. Mengali, M. Morelli, "Data-Aided Frequency Estimation for Burst Digital Transmission," *IEEE Transactions on Communications*, vol. 45, no. 1, pp. 23–25, Jan. 1997.

[7] D. C. Rife, R. R. Boorstyn, "Single-Tone Parameter Estimation from Discrete-Time Observations," *IEEE Transactions on Information Theory*, vol. 20, no. 5, pp. 591–598, Sept. 1974.

[8] ETSI, "Digital Video Broadcasting (DVB); Interaction Channel for Satellite Distribution Systems; (EN 301 790 V1.4.1)," Sept. 2005.

[9] ——, "Digital Video Broadcasting (DVB); Interaction Channel for Satellite Distribution Systems; Guidelines for the use of EN 301 790; (TR 101 790 V1.2.1)," Jan. 2003.

[10] S. Godtmann, N. Hadaschik, W. Steinert, A. Pollok, G. Ascheid, and H. Meyr, "Coarse and Turbo Synchronization: A Case-Study for DVB-RCS," in *Proc. of Joint NewCom-Acorn Workshop 2006*, Vienna, Austria, Sep. 2006.

[11] N. Noels, C. Herzet, A. Dejonghe, V. Lottici, H. Steendam, M. Moeneclaey, M. Luise, L. Vandendorpe, "Turbo Synchronization: An EM Algorithm Interpretation," in *Proc. of IEEE International Conference on Communications (ICC)*, vol. 4, Anchorage, Alaska, USA, May 2003, pp. 2933–2937.