Optimized Timing-Error-Detector for DS-CDMA Applications in Multipath Scenarios

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Abstract

An extension to the timing error detector (TED) for timing/code tracking loops used inside RAKE receivers in CDMA systems is presented. In contrast to the conventional TED it is well suited for the case of multipath propagation channels. In order to accomplish this task, the conventional early-late TED is replaced by an FIR filter which is optimized for the instantaneous multipath scenario. This optimization minimizes the overall distortions inside the timing tracking loop of each of the fingers of the RAKE receiver. The filter coefficients are calculated using the knowledge on the relative delays of all paths and their respective powers. An optimization like the one described here becomes necessary whenever closely spaced paths have to be tracked. This fact makes this algorithm a favorable candidate for indoor scenarios where individual paths can be spaced even more closely than one chip. The performance of the presented scheme is assessed by means of simulation.

1 Introduction

The timing error detector (TED) is a well known structure since the early days of digital communications. It has been used for transmission systems in the presence of flat fading for systems with and without spreading. The same structure has been employed for tracking individual paths in a multipath scenario. In doing so it has been usually assumed that the individual paths are well separated and the timing error detector for each individual path is not influenced by the presence of other paths. In general this is not true and in the case of closely spaced paths, as e.g. in the case of indoor scenarios this model mismatch becomes fatal and leads to significant performance degradations. In order to avoid these limitations of the conventional TEDs a new algorithm is presented that allows to reduce the effect of adjacent path interference on the TED. Several approaches for path delay tracking in frequency selective fading conditions have been proposed recently. A noncoherent tracking technique with interference cancellation was introduced in [3]. In [4], the author proposed a noncoherent scheme which jointly tracks a group of equidistant fingers. Lately a coherent cancellation scheme was presented independently in [5] and in [6]. A more simple low complexity version was presented in [7]. This paper is organized as follows: Firstly, the conventional approach is presented. Next, the multipath channel model for the next sections is presented. It is shown how the conventional approach suffers from severe degradations in this multipath scenario. In order to alleviate these degradations the EL-TED is replaced by a FIR. The filter coefficients of this FIR are optimized in a MSE sense. For this scheme, the achievable performance gains compared to the conventional scheme are demonstrated. The paper finishes with a conclusion and an outlook on some open topics.

*This work was supported by Agere Systems.

2 System Model

The system we are concerned with consists of a CDMA transmitter sending a complex valued data sequence $\{a_n\}$. These data symbols are spread by the spreading factor N using the effective spreading sequence $\{d_k\}$. This spreading sequence is assumed to be complex valued by itself. (The same overall technique as described here may be applied to different spreading schemes if the despreading scheme is adapted appropriately.) The spreaded sequence is transmitted using a pulse shaping filter $g(t)$ which in the case of 3GPP [9] is a root-raised cosine filter with a rolloff factor of 0.22. The resulting baseband-equivalent transmit signal is given by

$$s(t) = \sum_{k=-\infty}^{\infty} a_{\lfloor \frac{k}{N} \rfloor} d_k g(t - kT_c), \quad (1)$$

The signal from the transmitter travels through a multipath propagation channel with N_P independent paths (WSSUS model). Each of these paths is characterized by its delay τ

and channel coefficient $c^{(l)}$

$$h(\tau) = \sum_{l=0}^{N_p-1} c^{(l)}\,\delta(\tau - \tau^{(l)}) \tag{2}$$

As the signal enters the receiver white Gaussian noise $n(t)$ is added. In the first stage of signal processing in the receiver the signal is filtered by a filter matched to the pulse forming filter in the transmitter. In combination with the following RAKE [1] receiver a channel matched filter is formed. We constrain ourselves to this simple receiver type in order to show the important effects of multipath tracking that are common to all receivers relying on accurate timing estimates. The signal at the output of the pulse matched filter is given by

$$z(t) = \sum_{l=0}^{N_p-1} c^{(l)} \sum_{k=-\infty}^{\infty} a_{\lfloor \frac{k}{N} \rfloor} d_k R_g(t - kT_c - \tau^{(l)}) + \tilde{n}(t) \tag{3}$$

The noise term $\tilde{n}(t)$ represents both the noise filtered by the pulse matched filter and the interference by other users. Other user interference is modeled as additional noise in order to keep the model simple and highlight the important effects. The combined transmit and receive filter pulse form is denoted by $R_g(t)$, being the effective pulse form at the output of the pulse matched filter:

$$R_g(t) = \int_{-\infty}^{\infty} g^*(\tau) g(t+\tau)\, d\tau \tag{4}$$

The matched filter output signal is used as input for the RAKE receiver (see Fig. 1). The RAKE implementa-

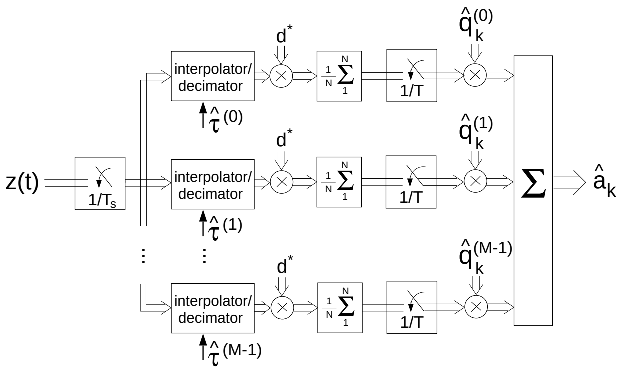

Figure 1. RAKE receiver model

tion shown here operates on samples from the matched filter output taken at an arbitrary rate $1/T_S$ (at least Nyquist sampling). From this point on, the implementation is fully digital. In each of the M branches of the RAKE the samples are processed by means of interpolation and decimation [2] in order to obtain intermediate samples with a rate of $1/T_C$ and to compensate for the estimated delay $\hat{\tau}^{(l)}$. Alternatively, instead of interpolation and decimation one could adapt the code phases in order to compensate the delays in the individual paths. Further operation is straightforward: The time compensated signal is correlated with the effective spreading sequence and accumulated over one symbol length. The signals from the individual paths are combined with appropriate weights q in order to obtain one estimate $\hat{a}$ at the output of the RAKE for each transmitted symbol. In the simulation section of this paper, maximum ratio combining based on estimates of the channel coefficients ($\hat{c}^{(l)}$) is used for determining the proper weights q. In order to be functional, a RAKE receiver needs estimates for the path delays ($\hat{\tau}^{(l)}$) and the channel coefficients ($\hat{c}^{(l)}$). The task of the timing/code tracker we are concerned with here is to keep track of changes in the $\tau^{(l)}$ during normal operation. Therefore, each branch of the RAKE has its own timing tracking loop.

3 Conventional TED

The conventional timing error detector often used in CDMA systems is the early late (EL) timing error detector. This timing error detector operates on two classes of samples of the matched filter output: one taken early and one late with respect to the detection path. For a good compromise between performance and implementation complexity the early and late branches are usually spaced half a chip apart ($T_c/2$) from the detection branch. In the case of no timing error everything is obviously balanced, hence resulting on average in no signal at the output of the timing error detector at all. In the case the signal is delayed by $T_c/2$ the late branch is perfectly aligned and therefore delivers a large positive output. The output of the overall timing error detector is calculated as the difference between late and early branch outputs. In the aforementioned case this leads to a positive value at the TED output for positive delays of the signal. In the case of negative delays with respect to the receiver time base the output becomes negative. The output of the conventional EL-TED is given by:

$$\begin{aligned} x_k &= x(kT) = Re\Bigg\{ \hat{a}_k^* \hat{c}_k^* \sum_{j=kN}^{(k+1)N-1} \\ &\Big(z(jT_c + T_c/2 + \hat{\tau}) - z(jT_c - T_c/2 + \hat{\tau}) \Big) d_j \Bigg\} \end{aligned} \tag{5}$$

If we assume a flat fading channel ($N_P = 1$) the output of the timing error detector, conditioned on the channel coefficient and dependent on the uncompensated timing error ($\hat{\tau} - \tau$), is on average given by

$$\begin{aligned} E[x|c] &= Re\Bigg\{ E[|a|^2]\,|c|^2 \Big[R_g(T_c/2 + \hat{\tau} - \tau) \\ &\qquad - R_g(-T_c/2 + \hat{\tau} - \tau) \Big] \Bigg\} \\ &= E[|a|^2]\,|c|^2\, S(\hat{\tau} - \tau) \end{aligned} \tag{6}$$

The conventional TED obviously operates correctly as expected in the case of a flat fading scenario. But in the case of multipath propagation ($N_P > 1$) the channel model changes, and therefore the output of the TED, conditioned on the set of channel coefficients $\mathbf{c}$, becomes strongly influenced by the additional paths:

$$E[x^{(m)}|\mathbf{c}] = E[|a|^2]\,Re\left\{ c^{*(m)} \sum_{l=0}^{N_p-1} c_l S(\hat{\tau}^{(m)} - \tau^{(l)}) \right\} \quad (7)$$

$$= E[|a|^2]\,Re\left\{ \left|c^{(m)}\right|^2 S(\hat{\tau}^{(m)} - \tau^{(m)}) \right\} \quad (8)$$

$$+ E[|a|^2]\,Re\left\{ c^{*(m)} \sum_{\substack{l=0..N_p-1 \\ l\neq m}} c^{(l)} S(\hat{\tau}^{(m)} - \tau^{(l)}) \right\} \quad (9)$$

The deteriorating effects described above do have a larger impact on the overall system than one might expect from their amplitude compared to the underlying noise processes of similar amplitude. This is the case because these effects originate from the bandlimited fading processes and therefore are band-limited to twice the bandwidth of the fading process. The bandwidth of this fading process, on the other hand, is usually much smaller than the overall signal and noise bandwidth, even for large mobile speeds. Therefore, the lowpass filter effect of the timing tracking loop filter will reject significant amounts of noise but cannot be as effective on the low frequency distortions in our case. For a visualization of system performance in the presence of uncompensated multipath distortions see Fig. 5. The term (7) not only consists of the desired term (8) as in the flat fading case (6), but it includes several additional terms (9). These additional terms can easily influence the signal at the timing error detector output in a way that it becomes completely useless. For each instance in time the output of the TED appears to be biased, depending on the present constellation of delays and channel coefficients of other paths. In the long term as the channel is assumed to be fading, this short term bias can be modelled as an increased estimator variance. But for low mobile speeds this approximation may be misleading, as, based on the biased estimate, the receiver may already have lost lock in the meantime. Therefore, averaging is not the method of choice in order to combat these effects.

4 New scheme

One possible extension to the conventional EL scheme is to use more than two samples in order to compute one TED value. A FIR filter may be used to accomplish this task. The FIR filter of length L is defined by its coefficients $\lambda = [\lambda_0 \cdots \lambda_{L-1}]$ and delays $\delta = [\delta_0 \cdots \delta_{L-1}]$ relative to the detection path. The expectation of the TED/FIR output, with respect to the channel fading coefficients, can be written as:

$$E[x^{(m)}|\mathbf{c}] = E[|a|^2]\,Re\left\{ c^{*(m)} \sum_{n=0}^{N_p-1} c^{(n)} \sum_{l=0}^{L-1} \lambda_l R_g(\delta_l + \hat{\tau}^{(m)} - \tau^{(n)}) \right\} \quad (10)$$

Therefore, the variance of the noise signal superimposed by path n on the TED output of path m, assuming a WSSUS fading, can be approximated ($\hat{\tau} \approx \tau, \hat{c} \approx c$) as:

$$\frac{1}{2}\left(E[|a|^2]\right)^2 \left| \sum_{l=0}^{L-1} \lambda_l R_g(\delta_l + \hat{\tau}^{(m)} - \tau^{(n)}) \right|^2 \ldots \quad (11)$$

$$E[|c^{(m)}|^2] E[|c^{(n)}|^2] \quad (12)$$

Obviously the amount of multipath distortion largely depends on the FIR coefficients that are used. In addition to this multipath distortion, there is still additive white Gaussian noise which is also filtered by the FIR. Assuming Nyquist pulses and a chip-spaced FIR, the attenuation factor for this channel noise is given by $\frac{1}{2}||\lambda||^2$. Nevertheless the two noise processes shown above have completely different impact on the performance of the TED in a closed loop structure. Depending on the loop bandwidth most of the channel noise is rejected by the loop filter. On the other hand most of the multipath interference passes the loop filter. Therefore, depending on the loop filter bandwidth and the spectral distribution of the noise processes, both noise terms have to be weighted appropriately.

5 Constrained optimization

For the new adaptive TED, one possibility of computing filter coefficients is to perform a constrained optimization, where the constraints are given by:

- zero crossing of S-curve at $\hat{\tau} = \tau$
- slope of S-curve normalized at $\hat{\tau} = \tau$

The task is now to minimize the MSE under these constraints. The result will depend on the length of the FIR filter, L, on the amount and location of the other paths and on the Doppler frequency spread. For general references on optimization, see [8].

In order to minimize the MSE a cost function $V(\lambda)$ is defined

$$V(\lambda) = ||\mathbf{A}\lambda||^2 + ||\lambda||^2 = \lambda^T(\mathbf{A}^T\mathbf{A} + \mathbf{I})\lambda \quad (13)$$

where the matrix $\mathbf{A}$ is $(N_p \times L)$ and contains the mean-square contribution of each path at each FIR filter tap location:

$$\mathbf{A} = \begin{pmatrix} A_{1,1} & \cdots & A_{1,L} \\ \vdots & \ddots & \vdots \\ A_{N_p,1} & \cdots & A_{N_p,L} \end{pmatrix}$$

$$A_{n,l} = \sqrt{\gamma\, w E[|c^{(n)}|^2]} R_g(\delta_l + \hat{\tau}^{(m)} - \tau^{(n)}). \quad (14)$$

γ is the overall SNR, with the coefficients c being normalized to this SNR. w is the appropriate weighting factor that reflects the different noise attenuation for multipath distortions and channel noise depending on the Doppler spread. τ and δ are normalized to the chip duration T_c. $\mathbf{I}$ is the $(L \times L)$ identity matrix. The aim is now to minimize $V(\lambda)$, subject to the constraints mentioned above:

$$\begin{aligned} \min_{\lambda} \ \{V(\lambda) \ : \ \mathbf{D}\lambda = f\} &= \min_{\lambda \in F} V(\lambda) \qquad (15) \\ F &= \mathbf{D}^+ f + N[\mathbf{D}] \end{aligned}$$

The matrix $\mathbf{D}$ is $(2 \times L)$ and contains the expression for the zerocrossing of the TED characteristic at $\hat{\tau} - \tau = 0$ and its slope at the same location:

$$\begin{aligned} \mathbf{D} &= \begin{pmatrix} D_{1,1} & \cdots & D_{1,L} \\ D_{2,1} & \cdots & A_{2,L} \end{pmatrix} \\ D_{1,j} &= R_g(t = \delta_j) \qquad (16) \\ D_{2,j} &= \frac{\partial}{\partial t} R_g(t) \Big|_{t=\delta_j} \qquad (17) \end{aligned}$$

If the slope at the origin is normalized to one, $f = [0 \ \ 1]^T$ describes the two conditions. $\mathbf{D}^+$ is the pseudoinverse of $\mathbf{D}$ and $N[\mathbf{D}]$ is its nullspace. The idea is now to move from a constrained optimization to an unconstrained one. To that end, the first step is to reduce the dimensionality of the problem. A singular value decomposition of $\mathbf{D}$ yields

$$\mathbf{D} = \mathbf{P}\,\mathbf{S}\,\mathbf{Q}^T, \qquad (18)$$

where $\mathbf{P}$ is (2×2) and $\mathbf{Q}$ is $(L \times L)$. The nullspace of $\mathbf{D}$ is equal to the range of a matrix $\mathbf{H}$, being defined as

$$\mathbf{H} = [q_{\nu+1} \cdots q_L], \qquad (19)$$

with the q_i being the indexed columns of $\mathbf{Q}$ starting from $\nu + 1$ with ν defined as the number of Eigenvalues. This identity yields the new unconstrained optimization problem

$$\min_{\theta \in R^{L-\nu}} V(\mathbf{D}^+ \mathbf{f} + \mathbf{H}\theta) \qquad (20)$$

Solving for θ finally yields

$$\theta = -\mathbf{C}^+ \mathbf{B} \qquad (21)$$

with

$$\begin{aligned} \mathbf{B} &= (2\mathbf{f}^T (\mathbf{D}^+)^T (\mathbf{A}^T \mathbf{A} + \mathbf{I})\mathbf{H})^T \\ \mathbf{C} &= 2\mathbf{H}^T (\mathbf{A}^T \mathbf{A} + \mathbf{I})\,\mathbf{H} \end{aligned}$$

The FIR filter λ is then given by $\lambda = \mathbf{D}^+ \mathbf{f} + \mathbf{H}\theta$. An example for a resulting S-curve in this case is shown in Figure 2. Here, 3 paths have been assumed at relative delays of $\tau_1 = 0, \tau_2 = T_c$ and $\tau_3 = 2T_c$, with root mean-square powers of 0 dB, -10 dB and -20 dB, respectively. It is easily seen that the contributions of the two paths adjacent to the one at the origin are nulled out by the zero-crossings of the S-curve at their respective locations.

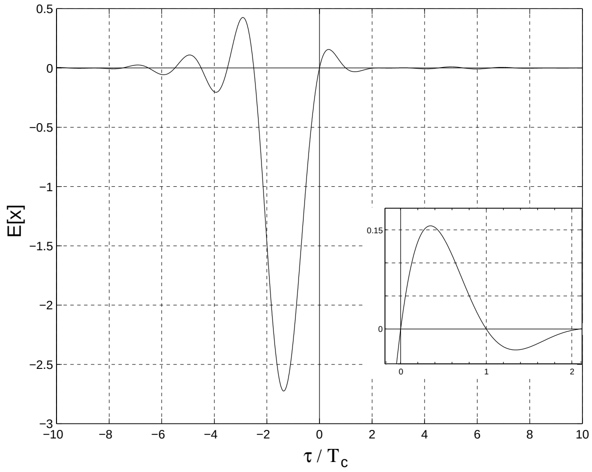

Figure 2. S-Curve for MMSE adaptively optimized FIR

6 Simulation Results

In order to demonstrate the achievable performance gains of the overall system, the new TED has been integrated into a tracking loop inside a RAKE receiver. The simulation setup was compatible with the 3GPP proposal [9]. An indoor scenario with spreading factor $N = 4$ and mobile speed $v = 10km/h$ was used. Fig. 3 shows the estimated delays of the two paths over a simulation interval of 1 second. The original paths are located at 0 resp. $1T_C$. If a conventional TED is used the weaker path moves towards the stronger one and merges within a fraction of a second. From that time on, only a combination of both paths is tracked. Even the stronger path is not tracked very well. If the adaptive FIR filter is used the crosseffects between

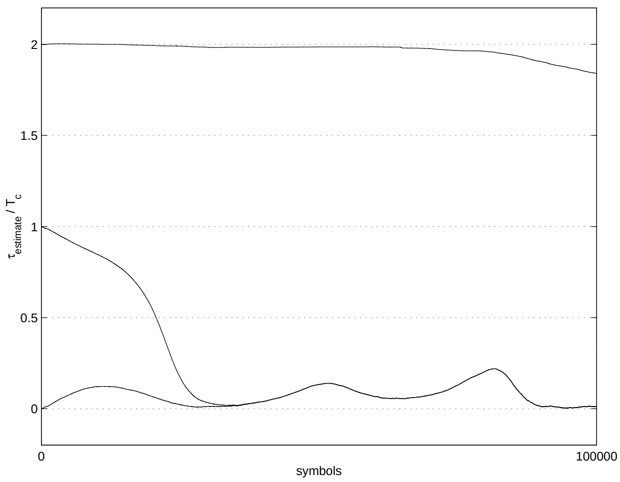

Figure 3. Timing tracking estimates, conventional TED, indoor, N=4, 10 km/h

the different paths are alleviated significantly and all paths are tracked almost perfectly (Fig. 4). Fig. 5 shows the resulting bit error rates for the scenarios described above. For the standard TED without compensation a BER degradation with resulting SNR losses as high as 5 dB can be observed. These losses are reduced to well below 1 dB if an adaptive FIR is used.

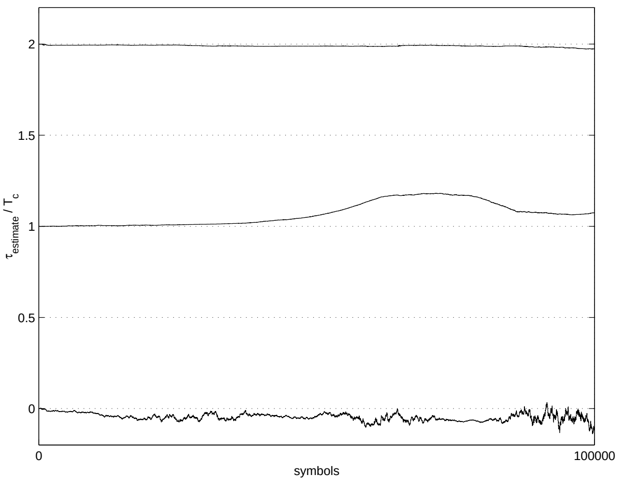

Figure 4. Timing tracking estimates, MMSE adaptive TED, indoor, N=4, 10 km/h

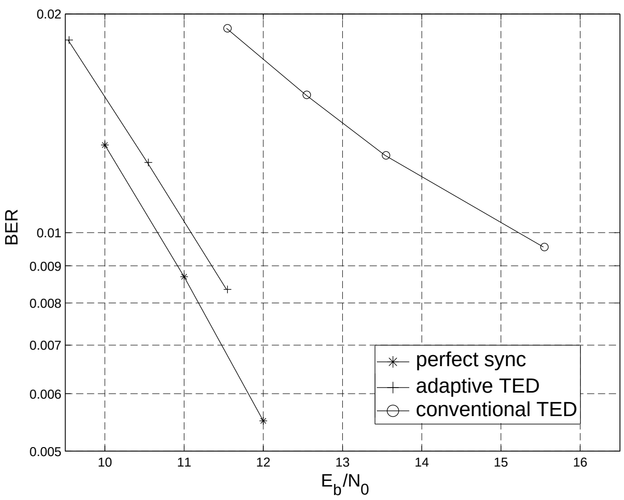

Figure 5. BER, adaptive code-tracking, N = 4

7 Conclusions and Outlook

The effects of multipath propagation on timing error detectors have been analyzed. The conventional EL-TED is covered as well as an extended version wherein the EL-TED is replaced by an FIR filter. The analysis provided the means for the optimization of FIR filter coefficients. Employing a TED extension as described in this paper extends the field of application for timing tracking loops to the area of closely spaced propagation delays as in the case of indoor scenarios. The achievable performance gain over the conventional TED without compensation schemes was illustrated by means of simulation. Field of further research will be a concise analytical treatise of the complete tracking loop in multipath scenarios. The effect of other users (synchronous and asynchronous) will be the topic of further studies.

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