# Performance Analysis of Phasor Estimation Algorithms for a FDD-UMTS RAKE Receiver

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*Abstract*— **The performances of phasor estimation algorithms for the UMTS RAKE Receiver are analysed. Due to the need of covering a wide range of possible mobile speeds and spreading factors, two different algorithms are considered here, namely a LMS-Kalman type algorithm, and a Wiener filter. The Wiener filter is considered both, as a predictor and a smother. It is shown that the simpler LMS-Kalman predictor is particularly suited for low mobile speeds and low spreading factors, whereas higher speeds and/or higher spreading factors require the usage of the more complex Wiener filter. Furthermore, the very high spreading factors (128-512) require the usage of the Common Pilot Channel in order to facilitate smoothing instead of prediction.**

## I. Introduction

The UMTS radio channel is characterized by multipath propagation where a number of reflected or scattered radio rays arrive at the receiving end. Each of the rays, as seen by the receiver, is characterized by a distinct phasor and time-delay. The RAKE receiver is suggested by ETSI [3] as a low-complexity solution for fast time-to-market, and will be the receiver of choice for the first wave of UMTS handsets. In each finger of the RAKE receiver, the amplitude, the phase shift, and the propagation delay of the selected multipath have to be compensated for, a task called synchronisation. The task of timing-error-detection is described, for example, in [4, 5, 6] which present a new class of high resolution algorithms. This paper is concerned with the description and performance analysis of phasor (phase and amplitude) estimation algorithms appropriate for the RAKE receiver. Any realizable receiver follows the concept of synchronised detection [1] for which a channel estimate $\hat{c}(\tau;t)$ or a sampled version thereof must be formed and subsequently used for detection as if it were the true known channel. Typically, the synchronisation algorithms consitute a significant portion of the overall computational complexity of the receiver, and the algorithm designer has the task to come up with a solution that offers good performance while spending a reasonable amount of computational effort. In the Downlink, the UMTS standard provides pilot symbols in the Dedicated Physical Control Channel (DPCCH) which is time-multiplexed with the Dedicated Physical Data Channel (DPDCH). Furthermore, there is provision for the Common Pilot Channel (CPICH) which employs a spreading factor of 256 and continously transmits pilot symbols. Phasor estimation is thus possible in a combined decision directed/data aided (DD/DA) scheme, which operates directly on the DPCCH/DPDCH data stream, or in a purely data aided (DA) fashion, exploiting the CPICH. The former method has the advantage that the CPICH doesn't have to be demodulated, thus saving computational effort in the receiver. Howver, since the DA/DD scheme relies on symbol estimates, the channel phasor has to be predicted. This apparent disadvantage can be avoided, if the CPICH is exploited, since then it becomes possible to employ a Wiener smoother, instead.

After reviewing a transmission model and the channel dynamics in Sections II and III, this paper describes phasor estimation algorithms appropriate for a RAKE receiver in the downlink IV. Finally, the performances of the algorithms are discussed in Section V.

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## II. Transmission Model

In a CDMA transmission system, the user data symbols $\{a_k\}$ are oversampled by the spreading factor $N_c = T/T_c$ and then multiplied by a user spreading sequence $\mathbf{d} = (d_0 \dots d_{N_c-1})$, $T$ and $T_c$ being the symbol and chip duration, respectively. The baseband-equivalent received signal $r(t)$ which is subject to multipath propagation is given by [1]

$$r(t) = \sum_{l=0}^{N_p-1} c_l(t) \sum_n a_n \sum_{\nu=0}^{N_c-1} d_\nu g_T(t - nT - \nu T_c - \tau_l) + m(t) \qquad (1)$$

where $g_T(t)$ is the transmit filter impulse response, $N_p$ is the number of discrete multipaths and the $\tau$'s are the corresponding propagation delays. The term $m(t)$ models the additive white gaussian noise (AWGN) and any interference from other users, which we assume here to be approximately white. Therefore, the noise $m(t)$ is simply characterized by its power spectral density $N_0$. Furthermore, $c_l(t)$ denotes the time-varying fading coefficient of the $l$-th multipath. The received signal is now fed into the RAKE receiver which consists of $N_p$ fingers which are subsequently combined in the RAKE combiner. Figure 1 shows a model of a single RAKE finger. We now go on considering the received signal $z_{l;k}$ in the $l$-th finger after receive matched-filtering, interpolation and decimation to chip rate. The interpolation process, which is performed for each multipath in the individually assigned RAKE finger, takes care of the timing delays $\tau_l$ and delivers samples at the correct time instance, such that the resulting raised-cosine function behaves like the Dirac Delta $\delta(t)$ and cancels out any dependency on previous or later symbols. Furthermore it is assumed that, since UMTS is a wideband communication system, the individual multipaths are well separable, and the effect of the other interfering paths on $z_{l;k}$ can be neglected. This is, of course, an idealizing assumption, and multipath interference cannot be ignored in many cases. How to handle multipath interference in the context of phasor estimation for a RAKE receiver is described in [7]. Therefore, we now write

$$z_{l;k} = a_n \cdot \sum_{\nu=0}^{N_c-1} c_{l;k} \cdot d_\nu \cdot g(kT_c - nT - \nu T_c) + m_{l;k} \qquad (2)$$

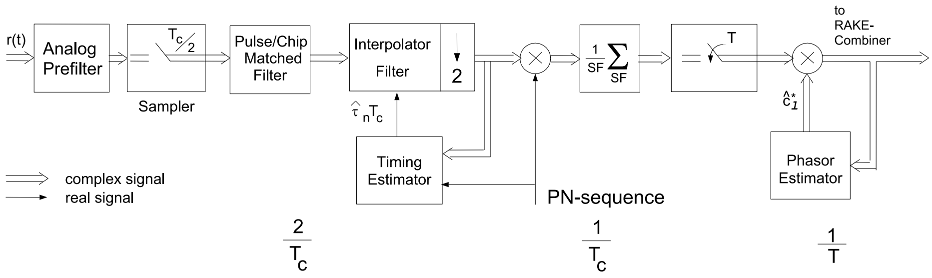


Figure 1: RAKE Finger

In the above equation, $g(t)$ denotes the raised cosine impulse response, which results from pulse-matched filtering the received signal with a root-raised cosine filter. The next step required for detecting the symbol is the despreading. In order to do so, the signal $z_{l;k}$ is now multiplied with the spreading sequence **d**, before summing the signal over one symbol period. Therefore, the despread signal, denoted with $y_{l;n}$, simplifies to an expression very familiar to the matched filter output known from traditional (non-spread) frequency nonselective single-carrier communication systems.

$$\begin{aligned} y_{l;n} &= \sum_{k=nN_c}^{(n+1)N_c-1} d_{k \ mod \ N_c} z_{l;k} + m_{l;k} \\ &= a_n c_{l;n} + m_{l;n} \end{aligned} \qquad (3)$$

Note, that the matched filter output sequences $\{y_{l;n}\}$ are dependent on the fading via the time-varying instantaneous SNR $\gamma_{l;n} = |c_{l,n}|^2/N_0$. Finally, exploiting the frequency diversity inherent in the received signal is accomplished by the combiner. The combiner forms a weighted sum of the signals $y_{l;n}$, by performing a linear combination of all $N_f$ assigned RAKE fingers.

$$\hat{a}_n = \sum_{l=0}^{N_f-1} q_{l;n} \cdot y_{l;n} \qquad (4)$$

with the optimal combiner coefficients $q_{l;n}$ given by

$$q_{l;n} = \frac{c_{l;n}^*}{\sum_{l=0}^{N_f-1} |c_{l;n}|^2} \qquad (5)$$

## III. UMTS Channel Dynamics

The time-variant fading coefficients $\{c_{l;n}\}$ are complex-valued random processes. The random fading process is assumed to be white-sense stationary (WSS), i.e., this process is sufficiently characterized by its mean and covariance. Furthermore, the $N_p$ fading processes are assumed to undergo mutually uncorrelated scattering (US), which is plausible since individual paths can often be attributed to distinct physical scatterers. The power spectral density $S_c(\psi)$ of a channel coefficient $c_{l;n}$ is given by the so-called Jakes spectrum. It is strictly bandlimited to the (normalized to symbol rate) Doppler frequency $\lambda_D'$ and its autocorrelation is denoted with $R_c$:

$$\lambda_D' = f_0 \cdot \frac{v}{c} \cdot T \qquad (6)$$

$$S_c\left(e^{j\psi'}\right) = \rho_l \cdot \begin{cases} \frac{1}{\pi\lambda_D'} \cdot \frac{1}{\sqrt{1-\left(\frac{\psi'}{2\pi\lambda_D'}\right)^2}} & \text{for } |\psi'| \leq 2\pi\lambda_D' \\ 0 & \text{otherwise} \end{cases} \qquad (7)$$

$$R_c(n) = \rho_l \cdot J_0(2\pi\lambda_D' \cdot n) = \rho_l \cdot \alpha(n) \qquad (8)$$

In the above equation, $\rho_l$ is the average process power of the $l$th path, $f_0$ is the carrier frequency ($2GHz$), $c$ is the speed of light, $v$ is the velocity of the mobile handset, and $J_0(\cdot)$ is the Bessel function of the first kind of order 0. UMTS foresees mobile velocities of up to $500km/h$. Considering the wide range of spreading factors from $N_c = 4$ up to $N_c = 512$, this results in a normalized Doppler frequency of $0.001 \leq \lambda_D' \leq 0.12$. The one extreme case of $\lambda_D' = 0.12$ translates into a maximum phase rotation of $43.2°$ *deg* per symbol, whereas for the other extreme case of $\lambda_D' = 0.001$ the phase rotation is negligibly small.

## IV. Phasor Estimation

Based on the transmission model given by equation (3), the optimal channel estimator is given by the conditional expected value given all available observations **y** and the corresponding symbol sequence **a**.

$$\hat{c}_n = E\left[c_n|\mathbf{y},\mathbf{a}\right] \qquad (9)$$

(For notational convenience, the subscript $l$ indicating the finger is omitted in this and the following Sections). In [1], it is shown, that this optimal channel estimate can be generated by low-pass filtering maximum-likelihood channel estimates $\hat{\mathbf{c}}_{ML}$ with a Wiener filter **w**, i.e. we have that

$$\hat{c}_n = \mathbf{w}^H \cdot \hat{\mathbf{c}}_{ML} \qquad (10)$$

Optimal Wiener filter theory requires that, for $N$ available observations, a Wiener filter of the same length be applied. Since $N$ is continously increasing during transmission, this would indicate the necessity of precomputing and storing many different sets of filter co-efficients. Evidently, this approach is not feasible, and one has to resort to reduced-complexity alternatives. Reduced-complexity channel estimation may be accomplished by either finite-length FIR or recursive IIR filtering. The former methods uses filter coefficients fixed to an arbitrary number $N_w$. The estimator therefore reduces to a single, time-invariant $N_w$-tap FIR filter $\mathbf{w} = (w_0, w_1, \ldots, w_{N_w-1})^T$. How to obtain the filter taps weights and error covariance is shown later on in Section IV. As mentioned before, the latter solution consists in recursive IIR-type filtering leading to the LMS-Kalman approach described in the following Section. In essence, it can be said that, both approaches rely on ML channel estimates, which are simply postprocessed (filtered) in different ways, by either an FIR or an IIR filter. The ML channel estimates are simply obtained by multiplying the observation with the complex conjugate of a known symbol, thus un-doing the modulation, i.e. $\hat{c}_{ML;n} = a_n^* \cdot y_n$ [1]. The resulting structure for channel prediction is illustrated in Figure 2. The required known symbols $a_n^*$ are either generated by forming hard decisions after the RAKE combiner, or by inserting known pilot symbols. The ML channel estimates are then filtered with either an FIR-type Wiener filter or with an IIR-type, so-called LMS-Kalman

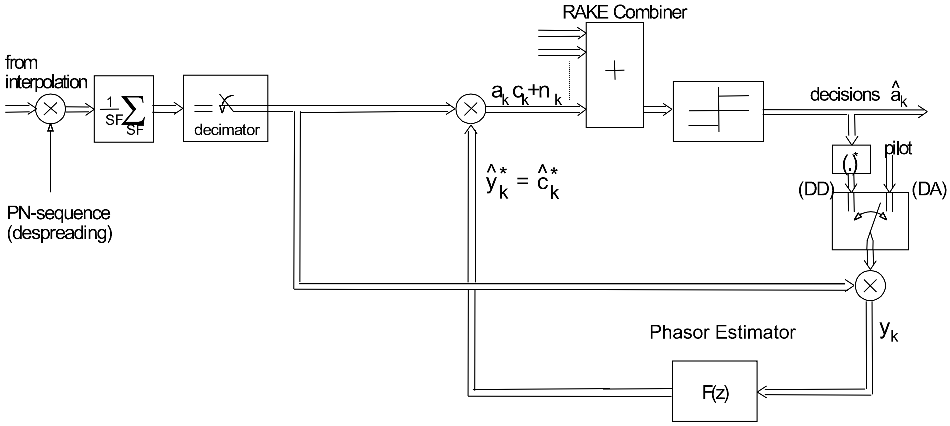


Figure 2: Phasor Prediction for a RAKE Receiver

filter. The corresponding filter is denoted in the illustration with $F(z)$. If Wiener smoothing using the Common Pilot Channel (CPICH) is considered, the structure shown in Figure 3 results. By changing the delay $z^{-P}$ between the Common Pilot Channel and the Data Channel, the quality of the Wiener smoother can be adjusted. Notice also, that the CPICH is always transmitted using a spreading factor of 256. If the Data Channel employs a lower spreading factor, the output of the Wiener Smoother has to be interpolated in order to deliver channel estimates at the correct rate which adds additional complexity.

### A. LMS-Kalman Channel Prediction

In this paper we consider using a LMS-Kalman filter for channel prediction. In order to keep channel prediction as simple as possible, let us constrain the Kalman filter to be a first-order, stationary IIR filter with scalar gain $K$. The recursive channel predictor then simplifies to [1]

$$\hat{c}_{n|n-1} = \hat{c}_{n-1|n-2} + K(\hat{c}_{ML;n-1} - \hat{c}_{n-1|n-2}) \tag{11}$$

This algorithm is identical in form with the well-known least-mean-square (LMS) adaptive filter with gain factor $K$. Notice, that this algorithm is only stable, if $K < 1$. In [1], it is shown that the optimal gain factor and the corresponding variance of the channel estimate can be closely approximated by

$$\begin{aligned} K^{(opt)} &\approx 3.6\left((\lambda_D')^2 \cdot \frac{\rho}{N_0}\right)^{\frac{1}{3}} \quad \text{s.t.} \quad K^{(opt)} < 1 \\ \sigma^2_{c;K} &\approx 12 \cdot \left(\frac{\lambda_D'}{K}\right)^2 + \frac{K \cdot N_0}{2-K} \end{aligned} \tag{12}$$

### B. Wiener Channel Prediction & Smoothing

The Wiener phasor estimator is identical in form to the linear minimum mean square error (LMMSE) [2] estimator. In [1] it is shown, that the estimator and the corresponding MMSE are given by

$$\hat{c}_n = \underbrace{\mathbf{c}_D^H \cdot (\mathbf{C}_D + N_0 \cdot \mathbf{I})^{-1}}_{\mathbf{w}^H} \cdot \underbrace{\begin{pmatrix} \hat{c}_{ML,n-N_w+P} \\ \vdots \\ \hat{c}_{ML,n-1+P} \end{pmatrix}}_{\hat{\mathbf{c}}_{ML}}$$

$$\sigma^2_{c;W} = \mathbf{w}^H(\mathbf{C}_D + N_0 \cdot \mathbf{I})\mathbf{w} - \mathbf{w}^H\mathbf{c}_D - \mathbf{c}_D^H\mathbf{w} + \rho \tag{13}$$

where

$$\mathbf{C}_D = \rho \cdot \begin{pmatrix} \alpha(0) & \cdots & \alpha(N_w-1) \\ \vdots & \ddots & \vdots \\ \alpha(N_w-1) & \cdots & \alpha(0) \end{pmatrix} \tag{14}$$

$$\mathbf{c}_D = \rho \cdot (\alpha(N_w-1-P) \cdots \alpha(1-P))^T \tag{15}$$

are the channel tap autocorrelation matrix and the vector of channel tap autocorrelation samples. The variable $P$ indicates, whether the estimator works as a predictor or a smoother. For $P > 0$, we have a Wiener smoother, which requires the knowledge of future ML phasor estimates. These future estimates can be obtained only, if a corresponding delay is introduced between the demodulator for the CPICH and the DPCCH/DPDCH data stream. Note, that the factors $\alpha(m) = J_0(2\pi\lambda_D' m)$ in equations 14 and 15, and the associated channel autocorrelation function, depend on the normalized Doppler shift $\lambda_D' = \lambda_D \cdot T$. Ideally, the mobile handset knows its velocity and selects a corresponding set of filter coefficients $\mathbf{w}$. However, this approach is not feasible, because firstly it is difficult to estimate the velocity, and secondly it would require the storage of many different sets of coefficients. With increasing velocity, the channel spectrum and the corresponding (low-pass) Wiener filter becomes wider. Therefore, in order to guarantee that no information loss occurs, all filter coefficients should be designed for the maximum speed envisaged by the corresponding spreading factor. As a consequence, if the true velocity is lower, the channel spectrum is narrower than that of the Wiener filter, and therefore the Wiener filter will not optimally suppress noise.

## V. Analysis and Conclusion

In this section, the performance of Wiener and LMS-Kalman phasor estimation algorithms are compared analytically. The performance of both schemes can be assessed via the variance of the channel tap estimate. Expressions for these variances are given in equations (12) and (13). The results are shown in Figures 4 - 8. These Figures show the variance of the channel tap estimates versus the mobile velocity. The analysis was performed using fixed values of $N_w = 15$ for the number of Wiener coefficients, $\rho = 1$, and $SNR = 10dB$ for the signal-to-noise ratio. For spreading factors $N_c < 32$, a design mobile velocity of $v_d = 120km/h$ was used, since movement at a higher

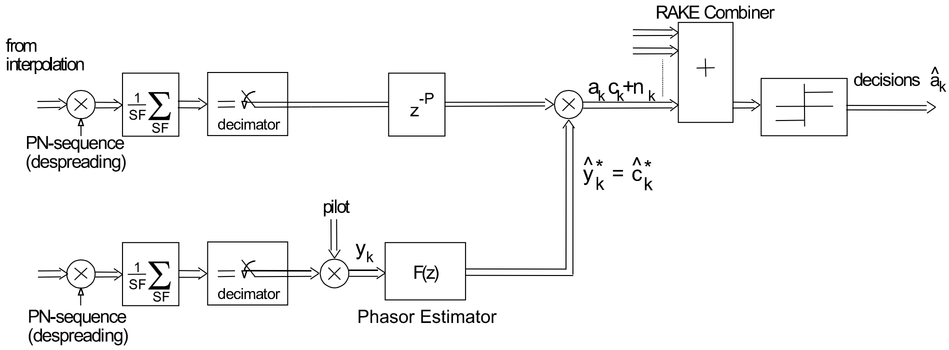


Figure 3: Phasor Smoothing for a RAKE Receiver

speed is not envisaged. For higher spreading factors $v_d = 500km/h$. In the legends, Wiener ext indicates Wiener prediction, and Wiener int indicates Wiener smoothing. The Wiener smoother was used with $P = 8$. When the spreading factor $N_c$ is smaller in the data channel than in the common pilot channel, the output of the Wiener smoother has to be interpolated in order to deliver estimates at the correct rate. Here it was assumed that interpolation is achieved by using a separate set of $N_w = 15$ coefficients for each interpolated value.

Clearly, for $N_c < 32$, the LMS-Kalman predictor performs better than the Wiener phasor estimation. For $N_c = 4$ and $N_c = 16$, the advantage of using the LMS-Kalman predictor, instead of using Wiener phasor estimation, translates into a SNR advantage of $\approx 0.2dB$ and $\approx 0.1dB$, respectively. Apart from showing better performance, the LMS-Kalman predictor offers the advantage of being computationally very efficient. The reason why Wiener phasor estimation performs poorly at these low spreading factors, is simply that the channel tap autocorrelation function is very long, and an FIR of length $N_w = 15$ is comparatively too short to approximate the optimal Wiener filter well. At spreading factors $N_c = 32, 64$, both the Wiener predictor and the LMS-Kalman predictor show approximately equal performance. The Wiener smoother, however, exhibits significantly better performance which translates into an SNR advantage of $\approx 0.5dB$ and $\approx 1dB$ for $N_c = 32$ and $N_c = 64$, respectively. For $N_c = 512$ this advantage increases up to $\approx 4dB$, indicating that the usage of the CPICH is well advised, here. Notice, that at the very high spreading factors (256, 512), it is not possible to design an adequate LMS-Kalman gain factor $K$, since the constraint $K < 1$ cannot be satisfied. In Figure 8, we also show the performance of the estimation algorithms that are matched exactly to the true velocity. It can be clearly seen that designing the Wiener predictor for a worst case speed of $500km/h$ drastically reduces its usefulness at lower speeds. In other words, knowledge of the velocity would enable the receiver to employ computationally less complex algorithms

The Wiener filter coefficients $\mathbf{w}$ and the LMS-Kalman gain $K$ have to be designed for a specific tap-SNR $\rho'_l = \rho_l/N_0$. It is not feasible to store coefficients for all possible values of $\rho'_l$, and therefore it is necessary to find a suitable subset of cofficients without compromising performance significantly. Fortunately, the performance of the algorithms is very insensitive to the design SNR, and it can be shown that having just two sets of coefficients is enought to oerate in a SNR-region $> 20dB$ with virtually no losses at all. Consequently, the LMS-Kalman and the Wiener filter, both require estimation of tap powers $\rho_l$ and the noise power $\sigma_m^2$, in order to choose a corresponding set of Wiener coefficients or Kalman gain in an on-line fashion.

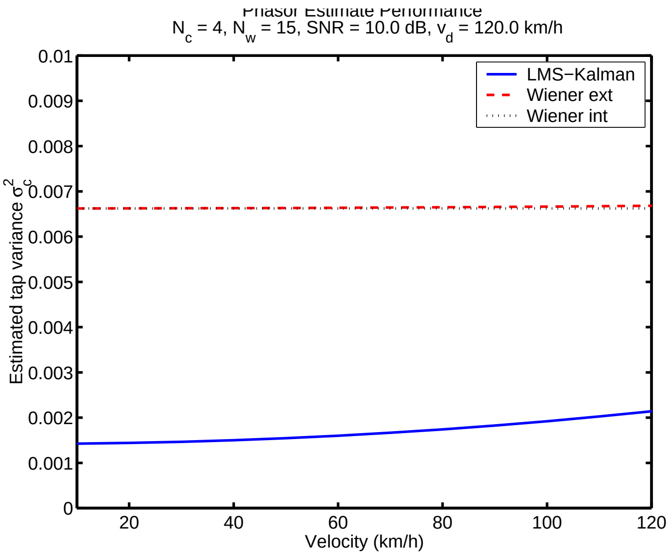


Figure 4: Phasor Estimate Performance – $N_c = 4$

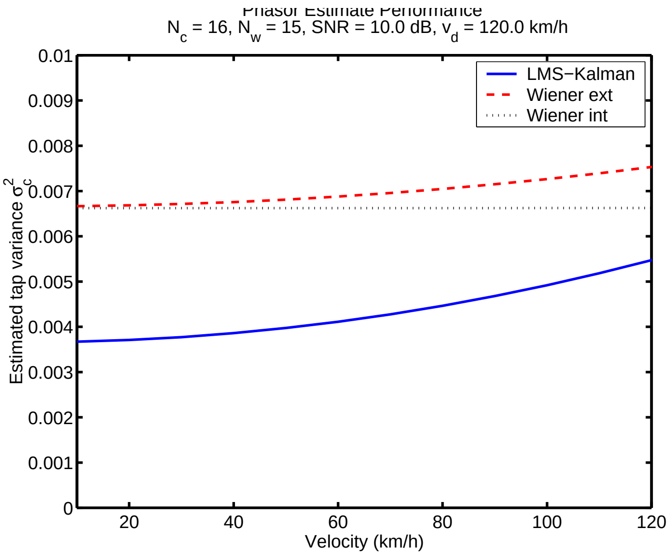


Figure 5: Phasor Estimate Performance – $N_c = 16$

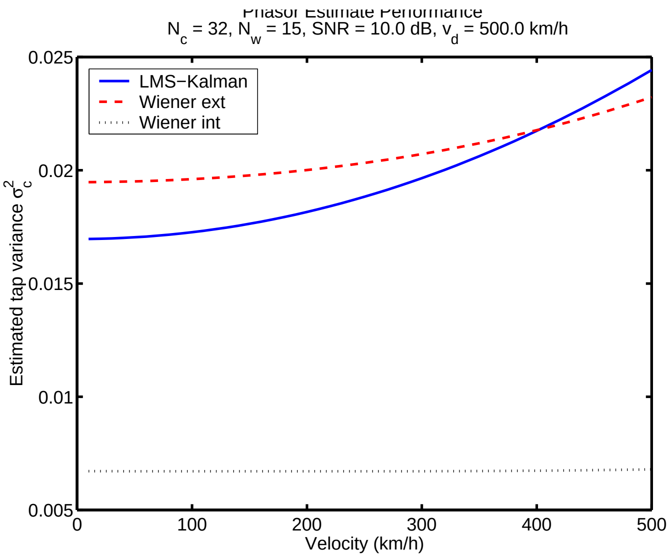


Figure 6: Phasor Estimate Performance – $N_c = 32$

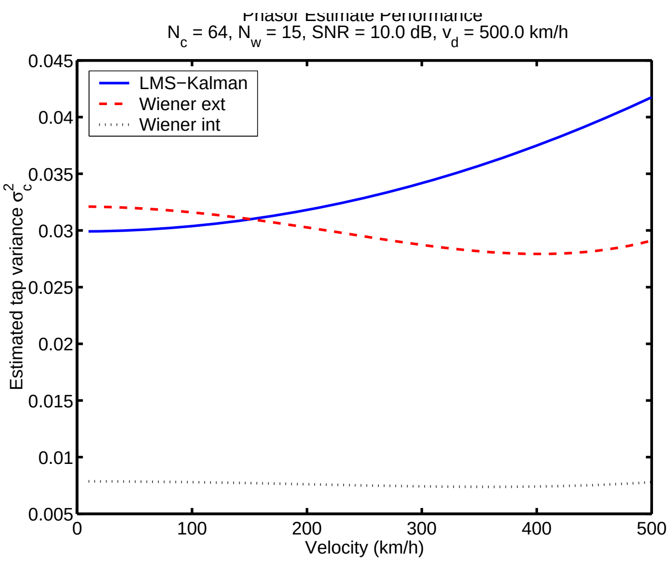


Figure 7: Phasor Estimate Performance – $N_c = 64$

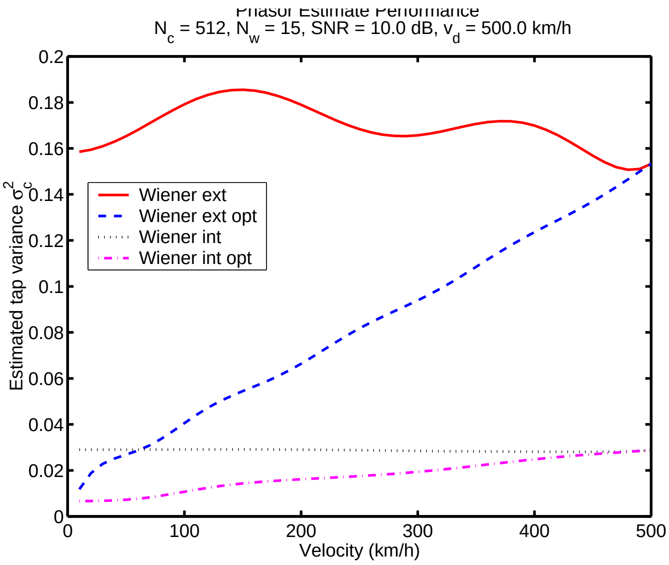


Figure 8: Phasor Estimate Performance – $N_c = 512$

## REFERENCES

[1] Heinrich Meyr, Marc Moeneclaey and Stefan Fechtel, *Digital Communication Receivers: Synchronization, Channel Estimation and Signal Processing*, John Wiley and Sons, New York, 1998.

[2] Steven M. Kay, *Fundamentals of Statistical Signal Processing – Estimation Theory*, Prentice Hall, Englewood Cliffs, New Jersey, 1993.

[3] 3GPP, *Technical Specification Group Radio Access Network; Physical layer - General description (3G TS 25.201 version 3.0.0.*

[4] G. Fock, P. Schulz-Rittich, J. Baltersee, H. Meyr. "Multipath Resistant Coherent Timing-Error-Detector for DS-CDMA Applications", ISSSTA2000, Newark, New Jersey, September 2000.

[5] G. Fock, P. Schulz-Rittich, J. Baltersee, H. Meyr. "Optimized Timing-Error-Detector for DS-CDMA Applications in Multipath Scenarios", submitted to Globecom2000, San Francisco, November 2000.

[6] P. Schulz-Rittich, G. Fock, J. Baltersee, H. Meyr. "Low Complexity Adaptive Code Tracking with Improved Multipath Resolution for DS-CDMA Communications ovdf Fading Channels", ISSSTA2000, Newark, New Jersey, September 2000.

[7] J. Baltersee, G. Fock, P. Schulz-Rittich, H. Meyr. "A Novel Multipath Interference Cancellation Scheme for RAKE Channel Estimation", submitted to Globecom2000, San Francisco, November 2000.