A Frequency Domain Variable Data Rate Frequency Hopping Channel Model for the Mobile Radio Channel

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Abstract—Modeling of the mobile radio channel is considered. Starting from a physical model, a frequency domain description of the channel is obtained. It is then shown that the effect of fading and dispersion can be entirely modeled using frequency transform techniques. Due to the bandlimitation of the simulated stochastic processes, the channel impulse response (CIR) model works on discrete time samples in both time and delay direction. Besides its computational efficiency an advantage of the technique is that the channel center frequency and the normalization unit (ie. the sample duration $T_s$) for the physical delay spread remain as generic parameters. The generic property of the model is its major advantage especially in the evaluation of new, unspecified scenarios. Due to the block–oriented processing, the fading-bandwidth–delay spread product is limited for this modeling technique. Applications to typical mobile radio data transmission scenarios conclude the paper.

Keywords—Fading Channels, Simulation, Radio Communication

I. INTRODUCTION

CHARACTERIZATION of the mobile radio channel is an old topic in communications and many approaches to model the complex behavior of the mobile radio channel have been taken in the past starting with [1] over [2] and [3], [4]. Each of these models was either dedicated to the classification of mobile radio channels or to the providing of a suitable simulation method. For many previously specified scenarios these models are nicely suited as all parameters important for the modeling process, especially the symbol rate, have been previously specified. In our paper, however, we focus on a mobile radio fading channel simulator allowing to specify the symbol rate as a parameter easily allowing to use one model setup for different symbol rate simulations and hence for an exploration of the communications system design space, where the symbol duration $T$, the symbol space order ($M$–QAM, $M$–PSK) and the FEC code rate have to be properly chosen.

Our paper is organized as follows: In section III, we describe the physical radio channel model our approach is based on. We comment on some statistical parameters and their restrictions for our model in section IV. Section V is devoted to the frequency domain representation of the channel model and section VI describes the FFT–based implementation of the model. Section VII discusses the application of the model and section VIII concludes the paper.

II. NOTATION

Some notation needs to be introduced first, so let $F_\omega$ be the fourier transform of a function with respect to the argument $x$ and $F^{-1}_\omega$ the respective inverse fourier transform with respect to $\omega$.

$$F_\omega f(x) = \int_{-\infty}^{\infty} f(x)e^{-j\omega x}dx \quad x \quad \text{real} \quad (1)$$

$$F^{-1}_\omega f(x) = \sum_{x=-\infty}^{\infty} f(x)e^{j\omega x} \quad x \quad \text{discrete} \quad (2)$$

The asterisk $*$ denotes complex conjugate. The operator $E_t$ denotes expectation with respect to $t$ and $\delta(x-x_0)$ is the dirac function where $\int a\delta(x-x_0)dx = a$. $X, Y, P$ are sets of integers.

III. THE PHYSICAL RADIO CHANNEL MODEL

We consider the transmission of digital data over a linear, time–variant channel at a radio frequency $\omega = \omega_B + p\Delta\omega$. The baseband channel model is described by its impulse response $c(t,\tau)$ that also may be seen as Green’s function of the corresponding linear time–variant differential equation. The output of the channel (ie. the solution to the inhomogeneous differential equation) is given by

$$r(t) = \int_{-\infty}^{\infty} c(\tau,t)s(t-\tau)d\tau \quad (3)$$

For any continuous–time $c(\tau,t)$ we can define a time–varying spectrum by

$$C(\omega_B + p\Delta\omega, t) = F_\omega c(\tau,t), \quad -\frac{\pi}{T_s} < \omega_B < \frac{\pi}{T_s}$$

$$P = \{-\infty, \ldots, 1, 0, 1, \ldots, \infty\} \quad (4)$$

where the limitation on $\omega_B$ will be used throughout the paper. For frequency hopping in the $p$th frequency slot, another (nonzero) center frequency is selected by choice of $p$. The channels are regularly spaced with a separation of $\Delta\omega$ and the baseband equivalent channel frequency response equals $C_p(\omega_B, t) = C(\omega_B + p\Delta\omega, t)$ with the new center frequency $\omega_B + p\Delta\omega$. For the evaluation of realistic scenarios, however, only a limited number of frequency slots $P = \{-P \ldots -1, 0, 1 \ldots P\}$ is of interest. Figure 1 depicts the selection of different frequency slots $p$ during the course of data transmission. As any realistic transmission system is either strictly band–limited(BL) or at least the overwhelming part of the signal energy is concentrated in a finite bandwidth interval (CPM systems[5]), we may sample the signal (and hence the channel $c$) at a suitable rate $1/T_s$ without significant (for BL systems: without any) error and we obtain

$$r(kT_s) = \sum_{l=-\infty}^{\infty} c(lT_s, kT_s)s(kT_s-lT_s). \quad (5)$$

(Sampling the bandlimited signal may be interpreted as the determination of the coefficients of the orthogonal base function set $s(lT_s - k)$.) For any practical system, only a finite number of taps $l$ must be considered. Also we might or might not want to include the effect of the transmit filter in the channel model. Now the channel impulse response is defined correctly for a bandwidth $1/T_s$. It is the goal of this paper to describe an efficient way of computing a finite–length $c^p(lT_s, kT_s)$ providing a means to apply frequency hopping (ie. variation of $p$) and variable symbol rates (ie. variation of $T_s$).
IV. SOME STATISTICAL PARAMETERS OF RADIO CHANNELS

The channel impulse response $c^p(t, \tau)$ may best be characterized by its correlation functions which come in four different well-known [1], [6], [7] flavors interdependent via Fourier transforms.

- spaced-time–spaced-frequency correlation function

$$R_C(\Delta \omega, \Delta t) = E_c \{ C(\omega, t) \cdot C^*(\omega + \Delta \omega, t + \Delta t) \}$$ (6)

- the spaced-time–delay correlation function

$$R_c(\tau, \Delta t) = E_c \{ c(\tau, t) \cdot c(\tau, t + \Delta t) \}$$ (7)

- spaced-frequency–Doppler power spectrum

$$S_C(\Delta \omega, \psi) = F_{\Delta \omega} R_C$$ (8)

- the delay–Doppler power spectrum (scattering function)

$$S_c(\tau, \psi) = F_{\Delta \omega} R_c(\tau, \Delta t)$$ (9)

where $\psi = 2\pi \lambda$ is the angular frequency of the Doppler spectra. See figure 2 for the relations of the functions introduced. Another important measure for the impulse response is the delay spread

$$\sigma_\tau^2 = E_c \int_{\tau=-\infty}^{\infty} \tau^2 c(\tau, t) d\tau -$$

$$- E_c \left( \int_{\tau=-\infty}^{\infty} \tau c(\tau, t) d\tau \right)^2.$$ (10)

The length of the delay profile is given by the interval $\Delta \tau$ containing all $\tau$ where the integral $\int_{\psi} S_c(\tau, \psi) d\psi$ is nonzero.

For a line-of-sight “scatterer”, the scattering function reads

$$S_c(\psi) = \delta(\psi - \psi_0).$$ (11)

Typically, the Doppler spectra of scatterers obey the Jakes spectrum

$$S_c(\psi) = \frac{2}{\psi_0} \frac{1}{\sqrt{1 - \psi^2/\psi_0^2}}.$$ (12)

where $\psi_0 = 2\pi f_D$ is the angular Doppler frequency.

V. CONTINUOUS–TIME FREQUENCY DOMAIN MODEL OF THE RADIO CHANNEL

A. Channel model without transmit filter

In order to derive our model, we partition the scattering function. We assume that the scattering function may be separated in the $\tau$ and $\psi$–direction and is independent of the transmission frequency slot $p$. Hence, we assume that the contribution of any scatterer is independent of the actual transmission frequency slot $p$. The only difference is that now the channel spectrum is evaluated at another center frequency.

$$S_c(\tau, \psi) = \sum_{i \in \mathcal{X}} S_{i}^q(\psi) \cdot q^i(\tau)$$ (13)

We assume further that the scatterers $i, j \in \mathcal{X}$ belonging to $S_{i}^q$ and $S_{j}^q$ are mutually uncorrelated and thus any correlation between different delays $\tau$ in $S_c$ is expressed by means of $q^q(\tau)$.

The spaced–frequency Doppler spectrum function now reads as follows

$$S_C(\omega, \psi) = \sum_{i} S_{i}^q(\psi) \cdot Q^i(\omega)$$ (14)

where $Q(\omega) = F_{\tau} q(\tau)$. We only want to model the CIR within $(-P - 0.5)\Delta \omega \leq \omega \leq (P + 0.5)\Delta \omega$. Hence any $S_C(\omega, \psi)$ can be written as a product of a rectangular window

$$\Pi(\omega) = \begin{cases} 1 & (-P - 0.5)\Delta \omega \leq \omega \leq (P + 0.5)\Delta \omega \\ 0 & \text{elsewhere} \end{cases}$$

and the right hand side of eqn. (14). Hence, $Q(\omega)$ is bandlimited. As we are interested in the representation of a single link only, we may further restrict the bandwidth of $\Pi$ to one frequency slot $p$

$$\Pi^P(\omega) = \begin{cases} 1 & -\pi/T_s \leq \omega - p\Delta \omega \leq \pi/T_s \\ 0 & \text{elsewhere} \end{cases}$$

Let us now distinguish between different cases constituted by different $q$.
1. The \( q^i \) consist of few dirac pulses at locations \( \tau_j \) where \( j \in \mathcal{J} \), only: Hence

\[
S_c(\tau, \psi) = \sum_j \sum_i S^i_{c}\delta(\psi - \tau_j) \tag{15}
\]

and

\[
S_C(\omega, \psi) = \sum_j \sum_i S^i_{c}\cdot q_{i,j}e^{j\omega\tau_j} \tag{16}
\]

Note that due to the band limitation of \( Q \) in (15) \( \delta(\tau - \tau_j) \) may be replaced by a \( \psi \) function with double sided bandwidth \( 2\pi/T_s \) since at any time instant, the CIR need only be known at a bandwidth of \( 1/T_s \) although the channel model altogether shall be valid for a larger bandwidth.

2. The \( q^i \) contain many dirac pulses or even continuous nonzero intervals. We may now exploit the band limitation of \( Q \) and hence sample \( q \) in the \( \tau \) direction with sampling time

\[
\tau_s = T_s \tag{17}
\]

Unfortunately, in the time domain, the result of the sampling process heavily depends on \( p \). In the frequency domain, however, the bandlimiting condition is expressed by multiplication with \( \Pi^p(\omega) \). In this case, the spaced–frequency Doppler spectrum function of interest reduces to

\[
S_C(\omega, \psi) = \sum_{j \in \mathcal{J}} \sum_i S^i_{c}\cdot q_{i,j}e^{j\omega\tau_j} \cdot \Pi^p(\omega) \tag{18}
\]

Obviously we will limit \( \mathcal{J} \) to be much smaller than the full infinite set of integers and take some approximation error into account.

In both cases, \( Q^i(\omega) \) is identical to a (infinite) sum of complex exponentials which can be truncated to a finite series. Letting \( \tau_j = j\tau_s \) for the second case and constraining our attention to \( \omega_B \),

\[
S_C(\omega_B, \psi) = \sum_{i \in \mathcal{I}} S^i_{c}(\psi) \sum_{j \in \mathcal{J}} q_{i,j}e^{-j(\omega_B + p\Delta\omega)\tau_j} \tag{19}
\]

can be used for both forms of \( q \). We may gather the correlations \( q_{i,j} \) in a positive semidefinite, factorizable[9] matrix

\[
Q = \Gamma^T\Gamma = (q_{i,j}) \tag{20}
\]

B. Transmit Filter

In order to take into account the transmit filter \( \sqrt{H(\omega_B)} \), equation (19) is multiplied with \( H(\omega_B) \), and the spaced–frequency Doppler spectrum now reads as follows

\[
S_C(\omega_B, \psi) = H(\omega_B) \cdot \sum_i S^i_{c}(\psi) \sum_j q_{i,j}e^{-j(\omega_B + p\Delta\omega)\tau_j} \tag{21}
\]

For consideration of CPM modulation, the transmit filter will be omitted. For \( P = 0 \), we also may include the TX filter in the computation of the correlation \( q^i(\tau) \) which in this case has to be convolved with the TX filter’s autocorrelation function.

The approach of choosing a discretization (by means of orthogonalization) was first proposed in [3], but, root–raised cosine pulses were chosen as base functions which did not permit the easy introduction of variable normalized tap delays and frequency hopping.

VI. THE DISCRETE–TIME CHANNEL MODEL

We describe the implementation of the channel model. All random processes involved may be sampled as we only deal with finite–bandwidth stochastic processes. The channel model is constructed by generating the respective discrete time stochastic processes:

1. Generate stochastic processes \( \zeta^i(kL_W T_s) \) with the respective scattering function \( S^i_{c}(\psi) \).
2. Compute their (correlated) sum \( \xi = \Gamma \zeta \).
3. Delay the resulting correlated processes \( \zeta^i(kL_W T_s) \) by \( \tau_j \) and sum them up in order to obtain the complete CIR. In case that the effect of the transmit filter shall be taken into account convolution (multiplication) with \( F^{-1}_n\sqrt{H(\omega_B)} \) (with \( \sqrt{H(\omega_B)} \) ) needs to be computed, too.
4. Filter the transmitted digital data with the resulting CIR.

A. Generation of stochastic processes \( \zeta^i(kL_W T_s) \)

The complex processes are generated from a gaussian source with variance \( \sigma^2_s \). The resulting complex gaussian process is then lowpass filtered in order to obtain a process with the power spectral density \( S^i_{c}(\psi) \). Any \( S^i_{c} \) from (11) and (12) is bandlimited, hence we may use a sampled input process (rate \( 1/M L_W T_s \)) without introducing errors. The transfer function of the filter has to approximate \( T(\psi) = \sqrt{S^i_{c}(\psi)} \) as the spectrum of the input gaussian process is flat. Many FIR filter functions produce a given power spectrum from given white input noise of which we select the real symmetric FIR filter. Let the length of the symmetric FIR filter corresponding to the spectrum \( \sqrt{S^i_{c}(\psi)} \) be truncated to \( L_i \). The periodic spectrum (period \( \pi/M^i L_W T_s \)) is sampled with \( N_r \) samples

\[
\psi_l = \frac{2\pi l}{M^i L_W T_s N_r}, \quad l = 0 \ldots N_r - 1 \tag{22}
\]

For a block–processing approach, for any input sample to be filtered correctly the past \( L_i - 1 \) FIR input samples must be present. Hence, the first \( L_i - 1 \) samples in each block have to be thrown away [10], [11], and, prior to the FIR block processing, samples have to be repeated. Figure 3 exhibits the basic structure of the overlap–and–save block processing. For very narrow bandwidths, the FIR output signal is interpolated by a rate \( M^i > 1 \) using linear or si–interpolation (\( M^i \) is an integer). Figure 4 displays the approximation of the Doppler spectrum produced by the frequency domain block processing filter.
Delayed the \( \xi^i \) is accomplished rather easily in the frequency domain: The spectrum (\( \omega_B \)-direction) of the undelayed \( \xi^i(\tau, kL_s T_s) \) is flat and thus

\[
\mathcal{F}_\tau \xi^i(\tau - \tau_j, kL_s T_s) = \mathcal{F}_\tau \left( \xi^i(0, kL_s T_s) \cdot \delta(\tau - \tau_j) \right) = \xi^i(0, kL_s T_s) e^{i(\omega_c \tau + p\Delta\omega)\tau_j}
\]

(24)

and the continuous–frequency spectrum of the channel is given by

\[
C(\omega_B, kL_s T_s) = \sqrt{H(\omega_B)} \sum_j \xi^i(0, kL_s T_s) e^{i(\omega_c \tau + p\Delta\omega)\tau_j}
\]

(25)

For further processing using the FFT, the spectrum (25) itself is represented by \( N_{C\text{IR}}/M_C \) samples

\[
\omega_{B,n} = \frac{2\pi n M_C}{N_{C\text{IR}} T_s}, \quad n = 0 \ldots N_{C\text{IR}}/M_C - 1
\]

(26)
in the interval \([0, 2\pi]\). For \( M_C > 1 \) (\( M_C \) is always an integer) interpolation (linear interpolation, for example) is used to increase the frequency sample rate by a factor \( M_C \). As the channel–impulse response is not strictly time–limited (but this is introduced by the model) even ideal interpolation will introduce additional errors. Hence, the vector

\[
C(n, kL_s T_s) = \sqrt{H \left( \frac{2\pi n}{N_{C\text{IR}} T_s} \right)} \cdot \sum_j \xi^i(0, kL_s T_s) e^{i(\frac{\tau_j}{N_{C\text{IR}} T_s} \tau + p\Delta\omega_j)}
\]

(27)

is finally used for representing the discrete–time channel impulse response, where depending on \( M_C \), at frequencies \( n \neq k M_C \) interpolated values may be used instead. In case that the transmit
filter is not taken into consideration

\[ C(n, kL_W T_s) = \sum_j \mathcal{E}_j(0, kL_W T_s) e^{j(n-kL_CIR) \frac{\tau_j}{\varphi_{\text{max}}}} \]

is used with the interpolated values replacing \( C(\cdot) \) depending on \( M_C \), again.

C. Filtering the transmitted samples with \( C(\omega_B, kL_W T_s) \)

The final step is filtering the transmitted data with the CIR. This is also performed in the frequency domain using the technique described in section VI-A and figure 3. The CIR is truncated to \( L_{\text{CIR}} \) samples. Obviously, the channel impulse response is now time–varying. Block processing cannot model the time–varying nature of the CIR without error. The reason is that the CIR is held constant for \( L_W = N_{\text{CIR}} - L_{\text{CIR}} + 1 \) data samples. But, as long as the sampling rate of the CIR, ie. \( 1/(L_W T_s) \), is much greater than \( \pi/\varphi_{\text{max}} \), the resulting approximation error is small and tolerable. At the price of increased computational effort, the final convolution may be carried out in the time domain allowing a per–sample adaptation of the CIR.

VII. APPLICATION OF THE MODEL

A. Model Parameters

The overall procedure of generating the complex, time–varying CIR is depicted in figure 5. In equations (28) and (27) the hopping frequency only affects the complex exponential representing the effect of the delay \( \tau_j \). A frequency hop can be applied at time instants which are multiples of \( T_s \), only.

The symbol duration is introduced in two positions: Firstly, it affects the normalized bandwidth of the process \( S_t(\psi) \) and secondly it affects the complex exponential expressing the delay as this contains \( T_s \), too. As for the power spectra of the fading processes in the relevant case (12) a closed from expression is available, it is very simple to parameterize the channel model with the symbol duration and a frequency hopping pattern. Only few spots of the model have to be adopted. For the LOS scatterer (11) a frequency domain filtering need not be computed but a multiplication with \( e^{j\psi_{\text{los}} kL_W T_s} \) can be carried out in the time domain, directly.

The physical model of the individual scatterers containing their power and their delay however is more elaborate and hence cannot be changed so easily per simulation run. For the application the model is dedicated to, however, this does not cause any difficulty as only the parameters for frequency hopping and the symbol duration (ie. \( p \) and \( T_s \)) were altered during the simulation.

B. Parameter Choice

We assume that a \( K \)-fold oversampling of the channel dynamics is sufficient for reducing the approximation error caused by the sample–and–hold block processing to a negligible amount. The maximum Doppler frequency is given by \( \varphi_{\text{max}} = 2\pi f_{\text{D}}^{\text{max}} \). The window for convolving the CIR with the transmitted data shall have a length of \( L_{\text{CIR}} = J \cdot \Delta T / T_s \). The length of the window with fixed CIR is then given by \( T_s L_W \). Hence, the CIR length–Doppler frequency product is limited by

\[ K f_{\text{D}}^{\text{max}} = \frac{1}{2T_s L_W} \]

For the slowly fading indoor channels with small delay spreads, higher accuracies can be achieved than for the outdoor, lower data rate channels. Reducing \( JK \) commonly means a gain in efficiency of the simulation.

In table I the setup for the three channels using the model discussed in here is described. An oversampling ratio of 2 with respect to the symbol (for the OFDM system: the reciprocal of the subchannel spacing) duration is assumed. For the DAB system, modeling is most crucial, but still, high oversampling ratios can be achieved.

VIII. CONCLUSIONS

We have presented a new method for the simulation of digital data transmission over the mobile radio channel. Our method allows easy reconfiguration of the symbol duration and usage of frequency hopping patterns. Implementation of the model is computationally efficient as fast convolution operations are used to speed up computing the result of FIR filter processes. Our model is limited to small products of fading bandwidth and maximum dispersion.

REFERENCES